ENERGY TRANSFORMATION IN A VORTEX TUBE

by

Stanley Kenneth Hutchin, Lt. USN

May, 1950

Thesis
H96

Thesis
H96

FASTENERS FOR BINDING
ACCOPRESS BINDER
No. BGS 2507
MADE BY
PRODUCTS INCORPORATED
N.Y. U.S.A

Library
U. S. Naval Postgraduate School
Annapolis, Md.

ENERGY TRANSFORMATION IN A VORTEX TUBE

By

Stanley K. Hutchin, Lt. USN

Submitted to the Faculty of Rensselaer Polytechnic Institute in partial fulfillment of the requirements for the degree of Master of Science.

May 1950

Thesis
H96

TABLE OF CONTENTS

Page

Summary
List of Symbols Used
Introduction 1
Equipment and Procedure
Theory 14
Results 22
Discussion 30
Conclusions 32
Recommendations 33
References
Sample Calculations
Figures and Tables
Calculation of Vortex Vane Settings Appendix A
Rotation of Air Without Vortex Tube Appendix B
Molecular Diffusion in Vortex Tube Appendix C

12835

LIST OF FIGURES

	Fig.
Schematic Diagram of Test Installation	1
Vortex Valve Assembly	2
Vortex Tube	3
Center Temperature	4
Wall Static Temperature	5
Radial Static Temperature Differential	6
Wall Static Pressure	7
Wall Total Pressure	8
Center Static Pressure	9
Wall Mach Number	10
Comparison of Wall Static Pressures to Theoretical Values	11
Wall Total Temperature	12
Radial Total Temperature Differential	13
Unmounted Vortex Valve and Tube	14
Mounted Vortex Valve, Tube, and Cone	15
Wall Probes and Cold Junction	16
Center Probe	17
Protractor Settings for Vortex Vanes	1-A
Vortex Valve Mounting	1-B

SUMMARY

The purpose of this investigation was to attempt to further the accumulated knowledge on the general aspects of air flowing at high angular velocities and under the influence of large viscous forces.

The air was introduced through a "vortex valve" to give it angular velocity and then spun down a pipe where various measurements could be taken.

In general it was concluded that a station of constant angular momentum would be difficult to attain and impossible to maintain in a moving medium and that heat transfer equilibrium, or constant static temperature in a radial direction, would not be reached.

SYMBOLS USED

Pc	Static pressure at the tube center.
Pw	Static pressure at the tube wall.
Po_w	Total pressure at the tube wall.
Po_p	Total pressure ahead of the vortex valve.
To_p	Total temperature ahead of the vortex valve.
To_w	Total temperature at the tube wall.
Tw	Static temperature at the tube wall.
Tc	Total temperature at the tube center.
Mw	Mach number at the tube wall.
ρ	Density of the air in slugs per cubic foot.
r	Radius of the vortex tube.
ω	Angular velocity of the air in the tube.
γ	$\frac{C_p}{C_v} = 1.395$
MV	Millivolt reading of potentiometer.
R	Gas constant = 53.35
V	$\omega(r)$

INTRODUCTION

The concept of the so-called forced vortex in an actual gas, as opposed to that of the classical free vortex having no viscous shearing forces, has been investigated quite widely in recent years. The refrigeration possibilities of such a flow were supposedly first recognized by a French scientist, G. J. Ranque who patented the Ranque tube in this country in the early nineteen thirties. During World War II, Rudolph Hilsch continued the vortex tube investigation hoping to provide refrigeration for certain German mines. Interest in this development was revived in this country in nineteen hundred and forty five and investigations have been conducted by numerous individuals and organizations since that time including: the General Electric Research Laboratory, DuPont de Nemours Research Laboratory, Pratt and Whitney Aircraft Corporation, Massachusetts Institute of Technology, and Rensselaer Polytechnic Institute. Although the explanations of the actual mechanism of the working of the Hilsch tube have been varied and contradictory, it has in general been concluded that it was unsatisfactory

for its original purpose, refrigeration, due to a comparatively low efficiency as a cooling unit.

Although the vortex tube proved to have no present application in the field of refrigeration, it is interesting to note that several practical applications of the vortex principle have been made. These include: commercial oil burners, dust cleaners, and more recently, a development by Dr. Vonegutt of the General Electric Research Laboratory of a vortex thermometer for reading not only the ambient air temperatures under adverse conditions of icing and turbulence but also the true air speed of a moving plane under the same conditions. There has also been some development by the NACA of axial flow compressors utilizing constant tangential velocity and constant angular velocity profiles to obtain greater blade tip loadings than were obtainable from the previous designs based on theoretical vortex velocity distributions.

Professor Neil P. Bailey, and members of the staff at Rensselaer Polytechnic Institute, proposed a theory that the forced vortex in an actual gas approached a condition of equilibrium such that the fluid mass rotated as a solid body. In this condition, the only viscous shearing forces present would be between the

fluid and the pipe wall and theoretically, the fluid should come to heat transfer equilibrium with a constant static temperature and a variable total temperature in the plane of constant angular velocity.

Several investigations were conducted at this institution on various phases of the forced vortex flow in an effort to learn as much as possible of the physical results of such a flow and the mechanisms responsible. W. C. Vickrey induced a vortex in a short two-inch tube at a low weight flow and finally concluded that a plane of constant angular velocity did exist. This was corraborated by L. M. Zubko working with greater weight flows and a four inch vortex tube. Ben B. Levitt then attempted an analysis of a vortex in a converging section with a sufficiently high weight flow to give supersonic flow. Levitt found no positive evidence that a station or region of constant angular velocity existed, and in none of these investigations was heat transfer equilibrium reached nor was it possible to maintain the condition of constant angular velocity over an axial distance greater than a thin plane.

It was the purpose of this investigation to turn to as small a vortex tube as practicable to instrument and use as high weight flows as could readily be

obtained with available equipment in order to produce a rotating system of extremely high angular velocity, pressure and temperature distribution, with a consequent better opportunity to determine more exactly some of the characteristics of the vortex type flow.

This investigation was conducted in the Mechanical Engineering Department, Rensselaer Polytechnic Institute, Troy, N. Y. during the period February to May, 1950.

The author wishes to express his gratitude to Professor Neil P. Bailey, Professor J. J. Devine, Associate Professor F. J. Bordt, Instructor J. F. Hill of the department staff and Ralph Giles, George Hudson and Waldo Goyer of the machine shop for their guidance, cooperation and patience.

EQUIPMENT AND PROCEDURE

A schematic diagram of the test set-up is shown in Figure 1 and photographs of the test installation and instrumentation in Figures 14, 15, 16, and 17. The compressed air source consisted of 2 six cylinder Schram compressors, Model 210, rated at 206 cubic feet of standard air per minute at 1175 RPM. A Westinghouse three phase, 60 cycle, 220 volt, 50 horsepower motor with a rated full load speed of 1175 RPM was connected by a direct drive to each compressor.

The air was led directly from the storage tanks to a plenum chamber using a coarse and fine control valve in parallel. A 3/4 inch area metering nozzle was located in the plenum chamber with pressure taps connected to a mercury manometer to give the pressure drop across the nozzle. A thermometer was installed downstream of the metering nozzle to measure total temperature ahead of the vortex tube. By knowing the pressure drop across the metering nozzle, its area and the total temperature, the weight flow for any given pressure drop could be calculated. Actually the weight flow was not directly involved in any of the vortex calculations and

the plenum chamber was used mainly for its utility in mounting and instrumentation.

The vortex plate, or vortex valve, was designed by Professor Bailey for another investigation but was modified slightly to fit the conditions of this development. The valve consisted of a 2-3/4 inch diameter, 3/16 inch thick brass plate on which were mounted 24 adjustable vanes secured on a 2-1/8 inch diameter circle. These vanes were set so they made an angle of ten degrees to the tangent of the circle on which they were mounted. This condition was chosen somewhat arbitrarily to give the best compromise between high angular velocities, low radial velocities, and high weight flow. Figure 2 gives the detailed specifications of the vortex valve and Figures 3 and 14 show its installation. The appendix to this report contains a development of the formula for setting the blade angles with an external protractor.

The vortex tube itself was of two inch inside diameter brass, 17 inches long with instrumentation holes every two inches starting ½ inch from the plenum chamber flange. It was necessary to build up the wall thickness at each of the instrumentation holes to provide greater rigidity for the instruments. Figure 3 contains

the detailed drawings of the vortex tube, and figure 15 is a photograph of the mounted tube on the plenum chamber.

The high angular velocities encountered in a vortex tube would be expected to give a radial pressure distribution which was higher towards the periphery than in the center. In this investigation this phenomena manifested itself in a sub atomospheric zone in the center. At the discharge end of the vortex tube there appeared to exist an unstable back flow condition into the low pressure central zone which resulted in an unbearable noise level. As this noise level definitely increased with weight flow, it was necessary to eliminate, or at least alleviate, the back flow before the desired high weight flows could be studied. The procedure used by Levitt was followed in this case with slight modifications. A two inch diameter aluminum cone, machined to a sixty degree taper, and rigidly mounted to a steel shank was placed at the discharge of the tube. The shank rode in a bushing which was mounted to the lathe bed, permitting the cone to adjust itself for the weight flow encountered. The cone and shank were both tapped for a .110 inch axial static pressure and total temperature probe. The operation of the cone was extremely

satisfactory at all weight flows, apparently eliminating all back flow and actually eliminating all of the noise. Figure 15 shows the cone slightly downstream of its proper position in order that the details may more readily be observed.

Both static pressure and total temperature probes were of .110 inch hollow steel wire 23 inches long. The static pressure probe was closed at its upstream end and drilled $\frac{1}{4}$ inch from this tip for a pressure tap. A brass nipple was soldered to the downstream end to hold the necessary rubber tubing.

The axial total temperature probe was made using an iron constantan thermocouple junction of number 30 wire, insulated from the enclosing steel tubing and welded at the tip and at the cold junction. A Leeds and Northrup potentiometer was used with a cold junction maintained at 32 degrees Fahrenheit in an ice-water bath. The thermocouple was intended for use only at the center of the vortex tube where it was expected that velocities would be negligible. All calibrations were then made under static conditions in a hot water bath through the range of temperature variation expected. No measurable variation was found between the thermocouple readings and an accurate standard mercury thermometer.

As it was mandatory that both axial probes be supported within the tube as well as by the discharge cone, a wire spider, soldered to a brass bushing, similar to the one used by Levitt, was tried. After repairing numerous spiders, it was decided that the design was not sufficiently strong to withstand the high angular velocities encountered and a support was constructed utilizing a thin outer brass shroud. As the entire support was approximately two inches downstream of the station under instrumentation, and the support was inserted for center measurements only, it was not felt that the support had any measurable effect on the actual static pressure or total temperature at the center. A photograph of the static tube and support is shown in Figure 17.

The wall total pressure and total temperature probes were designed by Zubko and for details of their design see Reference (a). The total pressure probe was modified slightly in that a space bar was installed to position the probe just clear of the boundary layer within the tube, and a pointer was mounted to indicate the direction of flow. The radial location of the probe was arbitrarily taken as that position which gave the greatest reading at station 5 and was approximately

.02 inches from the wall, clear of the boundary layer. The total temperature probe was calibrated at the discharge of a standard nozzle. It was assumed that the total temperature in the nozzle discharge was the same as the total temperature in the plenum chamber ahead of the nozzle. Several thermometers were used in the plenum chamber for a check on the total temperature and the thermocouple was also inserted into the plenum chamber to check against the reference thermometers. After checking the thermometers and the thermocouples against one another under the static plenum chamber conditions, air was forced through the nozzle up to a Mach number of one in the nozzle and under varying conditions of total temperature in the plenum chamber as the air increased in temperature. It was found that the total temperature probe in the discharge read temperatures that were comparatively as accurate as the other measured variables of the investigation. As no variation in total temperature of over three degrees Fahrenheit was discovered no corrections were made to the observed temperatures. The total temperature probe was, however, extremely directional in its characteristics, and would give correct readings only with the proper flow orientation. This position of correct reading was scribed on the probe, and when

readings were actually taken, the probe was rotated to select the correct position as indicated by the potentiometer.

The wall static pressure was obtained by a tapped and knurled brass plug, flush with the inner wall. Mercury manometers were used for both static and total pressures.

The pressure on the upstream face of the vortex valve did not enter into any of the calculations and was used merely to identify the runs, namely: 40 and 50 pounds per square inch gage. The axial stations on the tube were numbered from one to eight starting with the station nearest the plenum chamber.

The compressors were allowed to supply the vortex valve with the desired pressure until a steady total temperature was reached in the plenum chamber. This temperature varied from 112 degrees Fahrenheit to 118 degrees Fahrenheit on different days and it was necessary to correct all temperature readings to an arbitrary standard for comparison. It was found that normally the test installation had to be run for approximately an hour before the air temperature in the plenum had become sufficiently stabilized to permit readings to be taken. To completely stabilize this temperature, it was finally

necessary to provide increased ventilation of the laboratory; otherwise, the intake temperature of the compressors was constantly rising. As the compressors were set to unload at 100 psi gage, a bleed valve in the line was adjusted for each compressor at each weight flow to keep the compressors operating steadily.

In both wall and center temperature measurements, sufficient time was allowed between readings to allow the probes to reach a constant temperature as indicated by a constant reading of the potentiometer.

During actual operations, only the probe in use was inserted in the tube to minimize disturbances. The wall stations were first traversed for static pressure, total pressure, and total temperature. The center probes were not inserted until these traverses were completed and the hole through the discharge cone was covered when the center probes were not in use. After the wall traverses were completed, all of the wall instrumentation holes were plugged with special brass plugs and the center static pressure and total temperature were measured.

The point function relationships between Mach number and total to static pressure ratio and total to static temperature ratio were taken from the tables in

Reference (b) page 2.40. In using these tables, the ratio of total wall pressure to static wall pressure was first computed. Entering the tables with this value, the local Mach number at the wall and the ratio of total to static wall temperature could be obtained. From the temperature ratio, the wall static temperature could be computed.

THEORY

From the results of careful experimentation by this author and other investigators, there can be no doubt that energy transfer must take place in a vortex tube as evidenced by the high total temperature differential existing between the wall and the center of the tube.

For the moment, only the directed kinetic energy of the entering gas molecules will be considered. These molecules enter the vortex tube through the vortex valve with extremely high angular velocities and with comparatively low axial velocities. In general, energy transfer can occur by three separate mechanisms:

(1) Mass transport.

(2) Viscous forces.

(3) Conduction.

Assuming that all of the molecules entering the vortex tube do so with the same velocity and having the same directed and random energy, mass transport would not appear to offer any medium whereby energy could be abstracted from one molecule and added to another of

exactly the same energy.

Conduction of energy from molecules at higher temperatures to molecules at lower temperatures would tend to eliminate any existing temperature differentials and would surely provide no mechanism for giving a total temperature differential in the vortex tube.

This would appear to leave only viscous shearing forces as the mechanism whereby gases at the center of the vortex tube could give their directed kinetic energies to gases in layers further removed towards the periphery of the tube.

Considering at this time only the angular velocity components of the total flow velocity, it should be realized that viscous shearing forces are a function only of the angular velocity and not the tangential velocity of the gas.

Assuming that molecules enter the tube along definite streamlines, spiraling along definite stream tubes, and that the mechanism of filling the tube is one of an orderly progression along a streamtube rather than a random turbulence, the molecules that follow the stream tubes of decreasing radii will have a tendency to increase in angular velocity. This will of course lead to the establishment of angular velocity

gradients within the tube and will, at each successive radii, provide a mechanism whereby the molecules following the shrinking stream tubes can transfer some of their energy to the molecules immediately adjacent to them at greater radii. This process can continue until ultimately energy can be transferred from the inner or center molecules to the molecules at the periphery of the vortex tube.

The large radial pressure gradients existing in a vortex tube due to the high angular velocities, will give reasonably large radial static temperature gradients. These gradients must act in opposition to the previous effect, that is, to tend to eliminate any existing temperature differentials. There then exist the two opposing effects; flow of energy outward due to the viscous shearing action, and flow of energy inward due to thermal conduction.

Actually, all of the molecules entering the vortex tube will not have the same kinetic energy but will have energies corresponding to a statistical distribution curve. The higher directed kinetic energy molecules will then tend to follow the streamtubes along the wall of the tube and the lower directed kinetic energy molecules will tend to follow the streamtubes

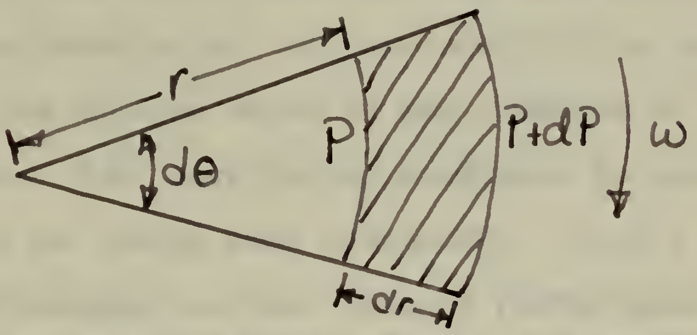
nearer the center of the tube, the exact tube followed depending upon the original kinetic energy of the molecule. This effect will in itself provide a total temperature differential, although there is no actual transfer of energy, only an apparent transfer.

In the following development it was assumed that there was no radial flow and that the axial flow at the center of the tube was negligible. The first assumption would be exceedingly difficult to either prove or disprove by instrumentation. As previously postulated, some radial flow must exist at the upstream end of the vortex tube to allow the tube to flow full when the air is expanded from the vortex valve. In general, it would appear that any radial component would be of negligible magnitude in comparison with the other velocity components.

A flow in angular rotation can be in pressure equilibrium, regardless of angular velocity distribution, if the physical conditions of the system permit axial velocity gradients. The true vortex flow is the only angular distribution which will give a uniform axial velocity. Any flow with an angular distribution such that the angular velocities near the center are less than those of a true vortex flow, will give higher axial

velocities at the periphery and lower axial velocities toward the center. See Reference (C)

From previous experimentation on this and similar apparatus, it is believed that the axial velocity was of sufficiently small magnitude to be ignored at the center of the vortex tube.



Assuming in a rotating fluid, with a differential element as pictured above, that there is no radial flow, the centrifugal forces of rotation must be balanced by the pressure forces on the element.

$$r\, dP\, d\theta = \rho\, r\, do\, dr\, \frac{r^2 w^2}{r}$$

$$dP = \rho\, dr\, r\, w^2 \text{ - - - - - - - - - - - - - } (1)$$

However:

$$\rho = \frac{P}{gRT} \text{ - - - - - - - - - - - - - - } (2)$$

Substituting (2) in (1):

$$\frac{dP}{P} = \frac{w^2\, r\, dr}{gRT} \text{ - - - - - - - - - - } (3)$$

Equation (3) is, of course, perfectly general for any angular velocity distribution and radial static

temperature distribution.

As previously stated, if the air flowing through the vortex tube contains high angular velocity gradients, viscous shearing forces will exist between adjacent layers of air at different velocities. It would be expected that the shearing forces between adjacent layers and between the outer layer and the wall would tend to bring the rotating air into an equilibrium condition in which the angular velocity was constant at any axial station. This equilibrium condition is commonly referred to as "solid body rotation". If this condition were attained, the only viscous forces present would be between the outer layer of the air and the wall of the tube, neglecting boundary layer effects. This force would tend to slow up the outer layers which would in turn tend to introduce slight angular velocity gradients in the tube and the process should again return to constant angular velocity at a lower value as the flow moves along the tube.

As there are no viscous shearing forces acting in the condition of solid body rotation, the air should come to heat transfer equilibrium with a constant static temperature throughout any radial cross section.

Assuming that such a station, or region, exists and that the static temperature were constant, equation (3) can be integrated to:

$$\ln(P_W - P_c) = \frac{w^2 r^2}{2gRT} = M_W^2 \frac{\gamma}{2} \quad ------ (4)$$

$$P_W = P_c \, e^{M_W^2 \frac{\gamma}{2}} \quad -------------(5)$$

This equation presents the wall static pressure in terms of the wall Mach number and the center static pressure under conditions of constant angular velocity and static temperature.

If the static temperature were not considered constant in a radial direction, but it was assumed that the temperature was a function of the static pressure such that:

$$T_W = T_c \left(\frac{P_W}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \quad -------------(6)$$

$$T_W = C_1 (P_W)^{\frac{\gamma-1}{\gamma}} \quad \text{where } C_1 = \frac{T_c}{(P_c)^{\frac{\gamma-1}{\gamma}}}$$

Substituting (6) in (3) and integrating:

$$\frac{dP}{P} = \frac{w^2 r \, dr}{gRC_1} \quad \frac{1}{(P)^{\frac{\gamma-1}{\gamma}}}$$

$$(P)^{\frac{\gamma-1}{\gamma}-1} \, dP = \frac{w^2 r \, dr}{gRC_1}$$

$$\frac{(P)^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}} = \frac{w^2 r^2}{2gRC_1} + C_2$$

$$(P)^{\frac{\gamma-1}{\gamma}} = \frac{\gamma-1}{\gamma} \frac{w^2 r^2}{2gRC_1} + C_3 \quad --- \;\; -----(7)$$

Solving for the boundary conditions to evaluate the constant of integration:

$$\text{at } r = o, \; C_3 = (P_c)^{\frac{\gamma-1}{\gamma}}$$

RESULTS

The data obtained by instrumentation is tabulated in Tables I and II. The data computed for the theoretical curves of Fig. 11 is tabulated in Tables III and IV. All computations used are either completely self evident or are demonstrated in the sample calculations at the end of this report.

Fig. 4 is a plot the variation of center total temperature, and center static temperature assuming no velocity at the center of the vortex tube, with axial station. Here the effect of the energy transfer from the center molecules to the outer molecules is plainly evident as center temperatures are as much as ninety five degrees Fahrenheit below the total temperatures existing in the plenum chamber. It should be noted that the lowest temperatures existant at the center occur at some distance downstream of the vortex valve, indicating that the energy transfer takes considerable axial distance. It can also be seen that the higher weight flow run gives a lower center temperature than the lower weight flow run due to the higher angular velocities and velocity gradients. The lowest temperature reached in the higher weight flow run occurs at a greater distance from the vortex valve than in the lower weight

flow run indicating, as would be expected, that the higher weight flow needs a longer time to dissipate its directed kinetic energy by viscous shearing forces. If the vortex tube were, as has been occasionally postulated, merely a device for separating high and low velocity molecules, there would be little difference in either minimum center temperature, or axial location of the point of minimum center temperature, with changing weight flow.

The rise in center temperature as the flow moves axially along the tube is of course due to an increase in the energy transfer inward due to conduction and a decrease of the energy transfer outward due to viscous forces.

Fig. 5 shows the variation in wall static temperature with axial station. In this figure it can be seen that the wall static temperature increases with increased distance from the vortex valve. This can be attributed to friction and turbulence converting total pressure into static temperature. No mechanism suggests itself for explaining the higher wall static temperatures at the lower weight flow run.

Fig. 6 shows the static temperature differential between the tube wall and center at the various axial locations. It can be seen that this static temperature

differential is nearly zero at the first station instrumented and might easily be zero at the vortex valve face. The static temperature differential then increases with increased distance downstream of the vortex valve, reaching a maximum at approximately eleven and one half inches from the valve for the lower weight flow run and at approximately thirteen and a half inches for the higher weight flow run. It becomes evident then, that wall static temperature and static temperature differential are a more direct function of the viscous forces than of the radial pressure gradients. It can be seen from this figure that the static temperature differential does tend to diminish but nowhere does it reach the condition of constant static temperature postulated by the equilibrium station in the theory. The rapid decrease at the last axial station should be viewed with suspicion as this is very near the tube discharge and it is felt that the discharge cone would have a decided effect on the flow at this point.

Fig. 7 shows the variation in wall static pressure with axial location along the vortex tube. This figure shows that the wall static pressure decreases in a regular manner with increased distance from the vortex valve. As the static pressure at the wall is a function

of the angular velocity and velocity gradients at that station, it cannot be stated exactly what the magnitude of this decrease would be in any given instance, not knowing the velocity distribution. In general, the over-all velocity of the rotating air should be decreased due to viscous forces which would tend to give a decreased pressure gradient. Assuming that the velocity distribution is practically the same at each of the weight flows, with the higher weight flow having the greater velocity at any radii, it is easily seen why the higher weight flow run would have the higher wall static pressure as indicated in Figure 7.

Fig. 8 shows the variation of wall total pressure with axial location along the vortex tube. As would be expected, the higher weight flow run gives the higher wall total pressure. The wall total pressure is dissipated quite rapidly due to the nature of the flow with its high viscous forces, and as was previously shown, appears as an increase in wall static temperature. Again, the greatest changes are visible nearer the vortex valve, indicating that the mechanism of energy transfer takes place principally in a finite distance from the vortex valve. If these curves in Fig. 8 were extrapolated back to zero axial location, the total

pressure would be very close to that in the plenum chamber, indicating very little loss in the actual expansion through the vortex valve.

Fig. 9 shows the center static pressure distribution with axial location. These curves are interesting in that they definitely do not corraborate the center traverses of either Vickrey or Levitt who found definite minimums in their traverses. These center traverses were checked quite carefully by this author and no indication of any pressure gradient was discovered. The sharp drop in static pressure near the discharge end of the tube is probably due to the center starting to accelerate under the disturbing influence of the back flow cone. The reasonably constant center static pressure also lends weight to the assumption that the center flow is negligible as there exists no mechanism for creating such a flow, particularly in the larger weight flow run. The smaller weight flow run shows an increasing pressure gradient with axial distance from the vortex valve which could indicate some backflow. As the cone to eliminate the back flow was arbitrarily designed for both weight flows on an approximate basis, rather than for one weight flow on an exact basis, some back flow could easily exist.

Fig. 10 shows, as expected, that the wall Mach number decreases with distance from the vortex valve. The highest Mach number obtained was approximately .890 and from the trend of the results, it is believed that extremely high plenum pressures would be necessary to increase this value significantly and the maximum weight flow used was approaching the limit of the vortex tube as designed.

Fig. 11 compares the actual wall static pressure distribution with the theoretical wall static pressure distributions based on the assumption that the angular velocity is at a condition of solid body rotation. This is of course done to ascertain if the flow actually does reach and remain at the constant angular velocity equilibrium condition. The theoretical curves are derived on both the ideal basis of no static temperature gradients and on the basis of the static temperature varying radially with pressure as if an isentropic compression had taken place. Actually, there is very little difference in the theoretical curves. It can be seen that nowhere do the theoretical curves approach the actual static pressure curve nor does there appear to be any tendency for the curves to approach one another. From this it appears that a condition of constant angular velocity

is not the preferential condition of the flow as postulated in the theory. The flow appears to follow some velocity pattern giving higher wall static pressures than constant angular velocity would allow. This pressure distribution was checked for several velocity distributions and appears to correspond to a velocity distribution part way between one of constant angular velocity and one of constant tangential velocity. Although the conditions as plotted are for the lower weight flow only, substantially the same results were found to exist at the higher weight flow condition.

Fig. 12 shows the variation of the wall total temperature with axial location along the tube. It is seen that at all locations, the wall total temperature is higher than the total temperature in the plenum chamber. As would be expected, the higher weight flow run had a larger total temperature than the lower weight flow run. It is interesting to note that the maximum wall total temperature occurs considerably closer to the vortex valve than the minimum center total temperature. This is probably due to the flow of heat due to static temperature gradients affecting the wall streamtubes sooner than it does the center section. There is also a possibility that increased radiation from the vortex tube

has some effect on the total temperature but with the high weight flows used this would appear to be a negligible factor.

Although the relatively slight increase in wall total temperature would appear somewhat at variance with the great total temperature drop at the center of the tube, it should be born in mind that the mass flow in a small annulus near the wall is equivalent to a large area near the center taking into consideration density gradients, velocity gradients, and diameters squared.

DISCUSSION

It becomes apparent from an analysis of the foregoing curves and from previous data taken by this author that the theory of the equilibrium or "solid body rotation" station is at varianco with the observed results. At first this author considered that the back flow cone that had been in reascnably general use in most of the development work at this institution forced a flow distribution that would not permit a constant angular velocity station. Experiments were conducted using a nozzle to eliminate the back flow, which was the antithesis of the cone, and in general the same results were obtained as those using the cone.

It was then seen that there existed another set of forces than those due to angular velocity gradients. From measurements of the direction of the flow at the wall at any axial station, it was concluded that there existed an axial component at the wall at station 1 of a Mach number of approximately .30 and at station 8 of .25. No axial components could be detected at the center of the tube and the axial distribution along any one radius appeared to be approximately linear. While these velocity gradients are not of the same magnitude as those of tangential components of the total velocity, they

appear to range from one third to one half the magnitude of the tangential or angular velocity gradients. The viscous shearing forces due to the axial velocity components can then no longer be ignored. From Reference (c) it is found that a rotating flow with a constant axial velocity can only be satisfied by a true vortex. Thus, it can be seen that the viscous shearing forces due to the axial gradients tend to give a constant axial velocity and a true vortex. In opposition, as previously stated, the viscous shearing forces due to the angular gradients tend to give a condition of constant angular velocity or "solid body rotation". These conditions are then incompatible and the equilibrium station as postulated in the theory section will not be obtained when high axial velocities relative to the angular velocities are encountered. It is possible that if high angular velocities could be obtained with low axial velocities, a condition of constant angular velocity could be reached but previous experimentation does not substantiate the retention of this angular velocity distribution over sufficient axial distance to permit heat transfer equilibrium to occur.

CONCLUSIONS

1. The vortex tube does actually provide for the transfer of energy from one molecule to the other although both molecules were originally at the same energy level.

2. The vortex tube is not merely a "molecule sorter" separating high velocity or energy molecules from low velocity ones, although this phenomena is present.

3. The transfer of energy in a vortex tube is accomplished by viscous shearing forces due to high velocity gradients.

4. The core of a vortex tube can deliver air at sub freezing temperatures.

5. The theory of a region of equilibrium at constant angular velocity and at constant static temperature is not valid due to axial velocities encountered in the actual vortex tube.

RECOMMENDATIONS

If the vortex flow is to be studied in the future with similar equipment, it is recommended that an annulus rather than a pipe be used. This would not only simplify the instrumentation but in any analytical development, would eliminate the inherent center discontinuity characteristic of most rotative flows.

Rather than using a vortex valve, the use of actual blades in the annulus to obtain the desired profile would be desirable. By this means, the original condition of the flow could be set and a mathematical development could possibly be made of any changes that occurred. In using the present vortex valve it is impossible to exactly define what the conditions are leaving the valve.

It is recommended that an investigation be made of the vortex tube as a gaseous diffuser of different weight molecules.

REFERENCES

(A) Zubko, L. M. "Characteristics of a Vortex in an Actual Gas." A graduate thesis prepared in the Department of Mechanical Engineering, Rensselaer Polytechnic Institute, Troy, N.Y., January, 1949.

(B) Bailey, N. P. "Thermodynamics of High Velocity Flow." A compilation of notes for course G 12:30 at Rensselaer Polytechnic Institute, 1949.

(C) Boltz, R. "Axial Flow Compressors." A compilation of notes for course G 1:12 at Rensselaer Polytechnic Institute, 1949.

SAMPLE CALCULATIONS

All data listed below was taken from Table 1, Station three (3).

$Pw = 14.70$ "Hg

$Po_w = 33.60$ "Hg

$Pc = -6.25$ "Hg

Barometer = 29.77 "Hg

$To_w = 586^{\circ}R$

$Tc = 484^{\circ}R$

- -

$Pw = 14.70 \quad 29.77 = 44.47$ "Hg

$Po_w = 33.60 \quad 29.77 = 63.37$ "Hg

$Pc = 29.77 - 6.25 = 23.52$ "Hg

$Po_w / Pw = 63.37 / 44.47 = 1.427$

Using the Mach number tables page 2.40 of Reference (B):

$Mw = .733 \qquad To_w / Tw = 1.106$

$Tw = 586 / 1.106 = 530^{\circ}R$

All temperature readings were originally taken in millivolts and directly transposed using Leeds and Northrup standard conversion tables.

To_p for the wall temperature readings was $118^{\circ}F$

To_p for the center temperature readings was $113^{\circ}F$

Po_p 50 psi gage

Barometer 29.77"Hg

Station	Pw "Hg	Pw "Hg	Po_w "Hg	Po_w "Hg
1	19.26	49.03	52.24	82.01
2	14.90	44.67	40.72	70.49
3	14.70	44.47	33.60	63.37
4	13.35	43.12	29.12	58.89
5	12.21	41.98	25.60	55.37
6	11.25	41.02	23.16	52.93
7	10.60	39.37	20.84	50.61
8	9.33	39.10	18.84	48.61

$To_p = 118°F$

$To_p = 113°F$

Tow(MV)	To_w °R	Tc(MV)	Tc °R	Pc "Hg
2.78	588	0	492	6.40
2.88	591	-.08	489	6.30
2.72	586	-.22	484	6.25
2.66	584	-.35	480	6.25
2.66	584	-.36	479	6.25
2.60	582	-.30	481	6.25
2.60	582	-.22	484	6.95
2.48	578	.50	509	11.30

Table I

Po_w/P_w	M_w	To_w/T_w	Tw °R	Pc
1.674	.892	1.161	506	23.54
1.579	.836	1.138	519	23.64
1.427	.733	1.106	530	23.69
1.368	.685	1.092	535	23.69
1.320	.644	1.082	539	23.69
1.290	.615	1.074	541	23.69
1.286	.611	1.073	542	22.99
1.240	.564	1.063	544	18.64

Table I (cont.)

Po_p 40 psi gage

Barometer 29.77"Hg

Station	Pw "Hg	Pw "Hg	Po_w "Hg	Po_w "Hg
1	14.75	44.52	42.70	72.47
2	12.26	42.03	32.50	62.27
3	11.26	41.03	24.48	54.25
4	10.64	40.41	23.76	53.53
5	9.99	39.76	19.52	49.29
6	8.36	38.13	18.26	48.03
7	8.35	38.12	16.38	46.15
8	7.55	37.32	14.84	44.61

$To_p = 120^\circ F$			$To_p = 112^\circ F$	
To_w(MV)	To_w $^\circ R$	Tc(MV)	Tc $^\circ R$	Pc "Hg
2.65	583	.24	500	6.25
2.84	590	.12	496	5.80
2.70	585	-.15	487	5.80
2.70	585	-.09	488	5.55
2.62	582	-.08	489	5.55
2.62	582	0	493	5.45
2.59	581	.10	496	6.00
2.59	581	.70	516	10.90

Table II

Po_w/P_w	M_w	To_w/T_w	Tw	Pc
1.626	.863	1.147	509	23.69
1.483	.773	1.118	528	24.14
1.322	.645	1.082	541	24.14
1.324	.648	1.083	540	24.39
1.241	.565	1.063	548	24.39
1.260	.585	1.067	546	24.49
1.209	.529	1.055	551	23.94
1.194	.510	1.051	552	19.04

Table II (cont.)

40 psi

$$Pw = Pc\ e^{\left(\frac{Mw^2}{2}\right)}$$

	Mw	Mw^2	Pc	$.700Mw^2$	$e^{.70Mw^2}$	$Pce^{.70Mw^2}$
1.	.863	.742	23.69	.519	1.681	39.8
2.	.773	.595	24.14	.417	1.517	36.6
3.	.645	.414	24.14	.290	1.337	32.2
4.	.648	.418	24.39	.293	1.341	32.7
5.	.565	.318	24.39	.223	1.250	30.5
6.	.585	.341	24.49	.239	1.270	31.1
7.	.529	.278	23.94	.1945	1.215	29.1
8.	.510	.259	19.04	.1814	1.199	22.8

50 psi

	Mw	Mw^2	Pc	$.700Mw^2$	$e^{.70Mw^2}$	$Pce^{.70Mw^2}$
1.	.892	.790	23.54	.553	1.738	40.8
2.	.836	.695	23.64	.487	1.627	38.4
3.	.733	.535	23.69	.375	1.456	34.4
4.	.685	.467	23.69	.327	1.387	32.9
5.	.644	.413	23.69	.289	1.336	31.6
6.	.615	.377	23.69	.264	1.302	30.8
7.	.611	.372	22.99	.261	1.298	29.8
8.	.564	.317	18.64	.222	1.249	23.3

Table III

$$Pw = Pc\left[M^2\,\frac{\gamma}{2}\,\frac{Tw}{Tc}\,\frac{\gamma-1}{\gamma} + 1\right]^{\frac{\gamma}{\gamma-1}}$$

40 psi

		A	B	C	D	E	
	Pc	$M^2/2$	$\frac{Tw}{Tc}$	.283(B)	C+1	$D^{\frac{\gamma}{\gamma-1}}$	PcE
1.	23.69	.519	1.013	.149	1.149	1.633	38.7
2.	24.14	.417	1.037	.1225	1.1225	1.504	36.3
3.	24.14	.290	1.102	.0905	1.0905	1.358	32.7
4.	24.39	.293	1.092	.0908	1.0908	1.360	33.1
5.	24.39	.223	1.093	.0693	1.0693	1.267	30.9
6.	24.49	.239	1.084	.0735	1.0735	1.285	31.5
7.	23.94	.1945	1.077	.0593	1.0593	1.226	29.3
8.	19.04	.1814	1.048	.0549	1.0549	1.208	23.0

50 psi

		A	B	C	D	E	
	Pc	$M^2/2$	$\frac{Tw}{Tc}$	.283(B)	C+1	$D^{\frac{}{-1}}$	PcE
1.	23.54	.553	1.017	.1592	1.1592	1.66	39.1
2.	23.64	.487	1.046	.144	1.144	1.607	38.0
3.	23.69	.375	1.062	.1128	1.1128	1.457	34.3
4.	23.69	.327	1.086	.1008	1.1008	1.403	33.3
5.	23.69	.289	1.098	.090	1.090	1.353	32.1
6.	23.69	.264	1.103	.0826	1.0826	1.323	31.3
7.	22.99	.261	1.098	.0813	1.0813	1.318	30.3
8.	18.64	.222	1.050	.0661	1.0661	1.253	23.3

Table IV

SCHEMATIC DIAGRAM OF TEST INSTALLATION

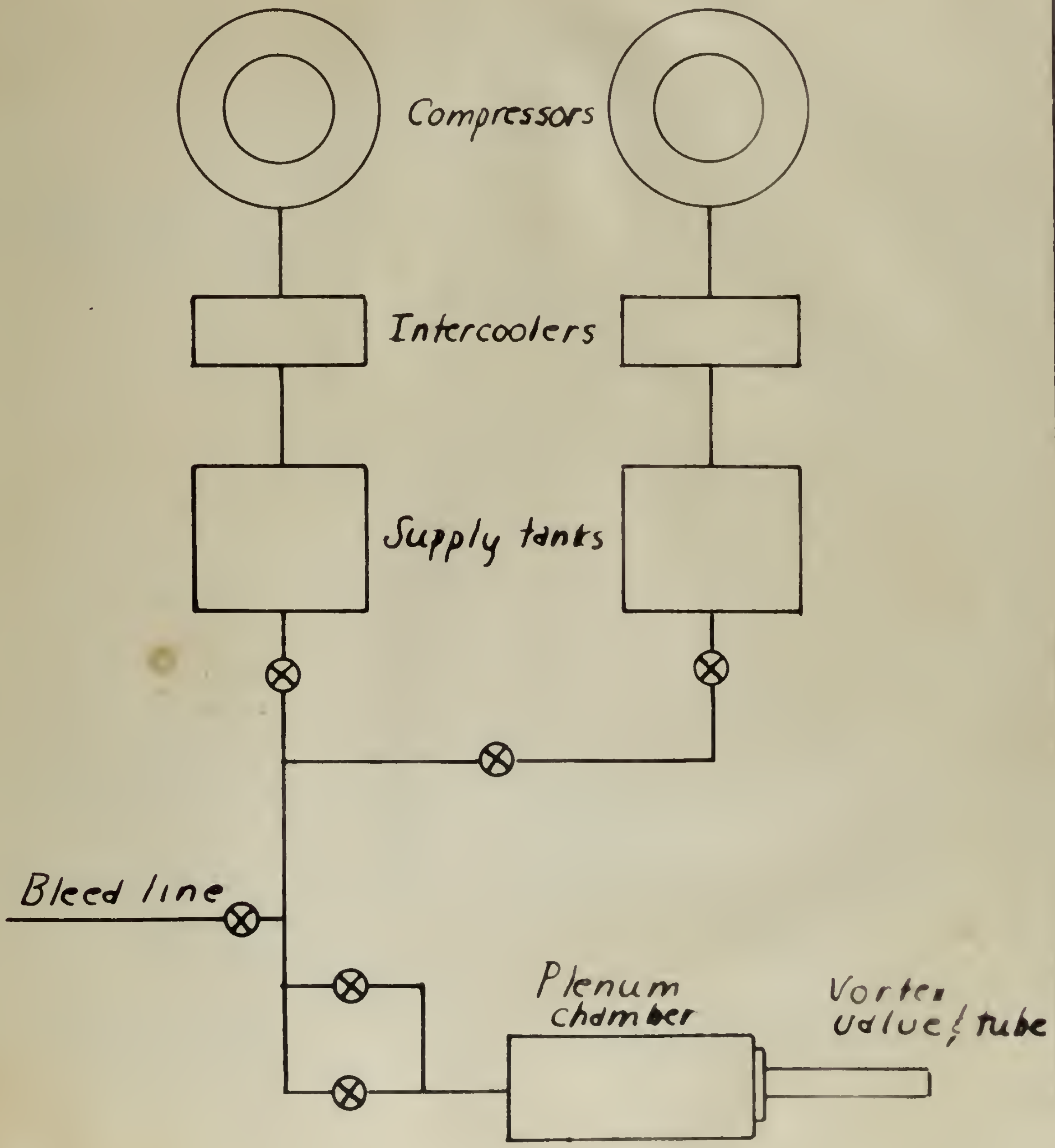


FIG. 1

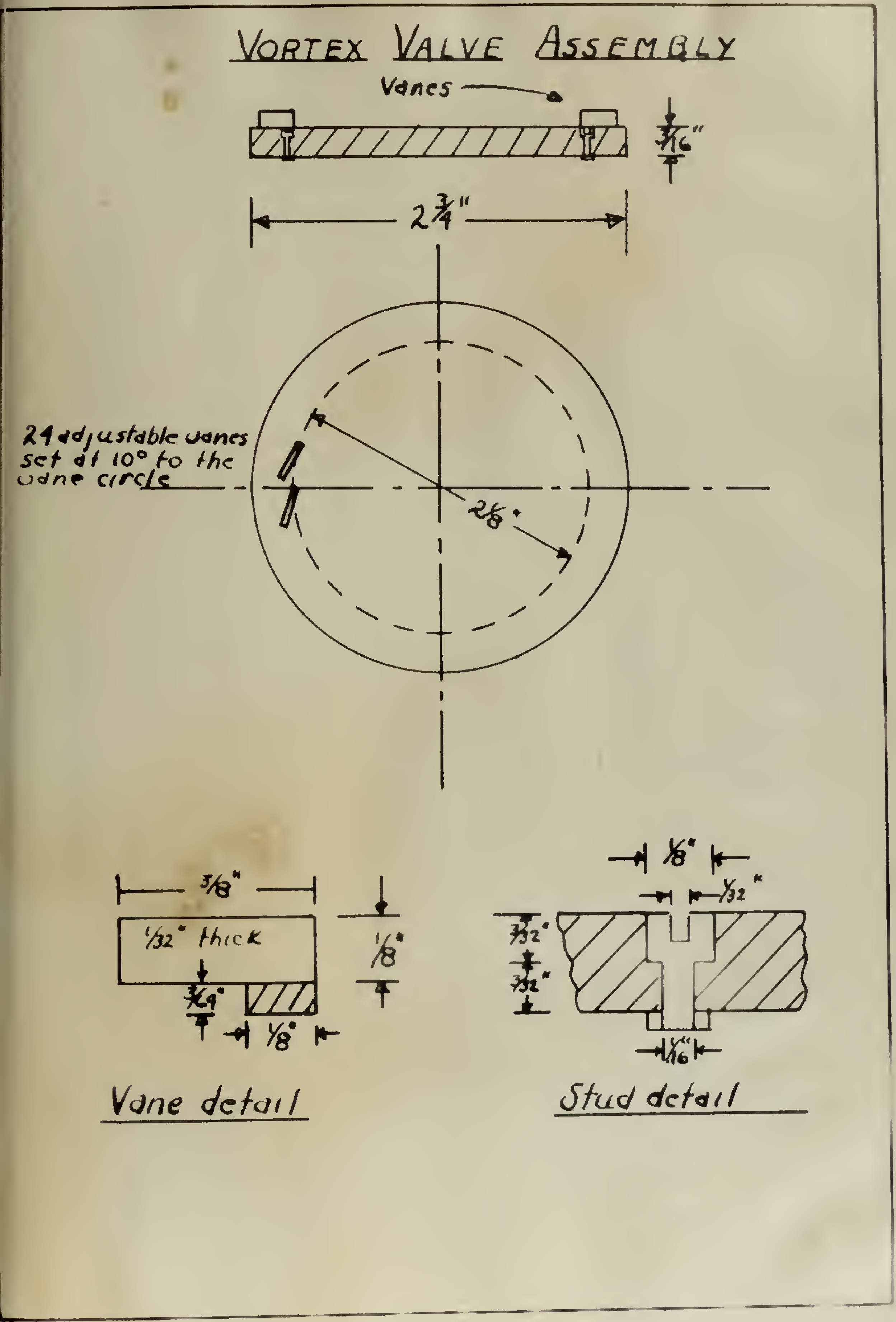

VORTEX VALVE ASSEMBLY
Vanes
3/16"
2 3/4"
24 adjustable vanes set at 10° to the vane circle
2 1/8"
3/8"
1/32" thick
1/8"
3/64"
1/8"
Vane detail
1/8"
1/32"
3/32"
3/32"
1/16"
Stud detail

FIG. 2

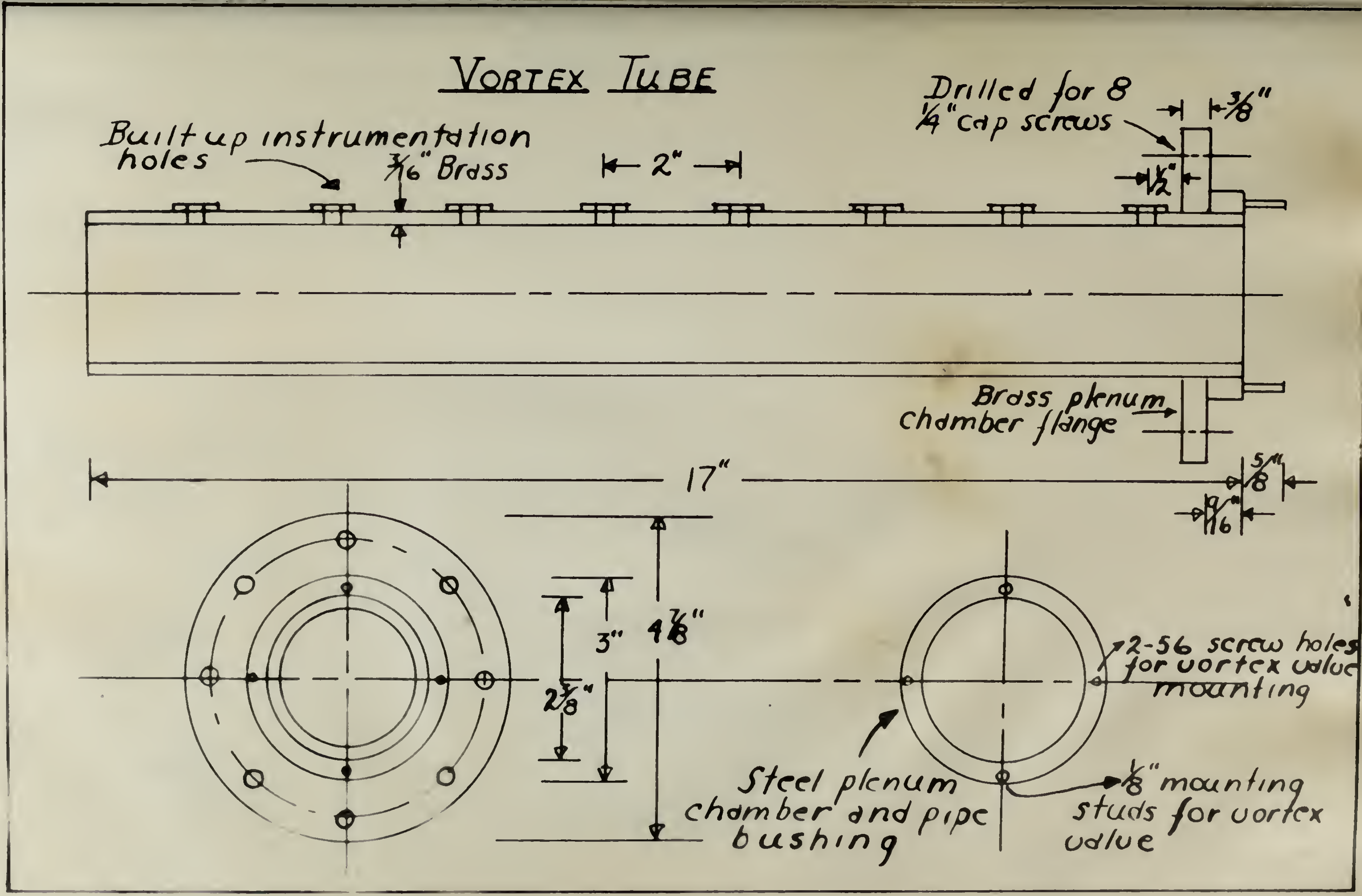

VORTEX TUBE
Built up instrumentation holes
3/16" Brass
2"
Drilled for 8 1/4" cap screws
3/8"
1/2"
Brass plenum chamber flange
17"
5/8"
9/16"
4 7/8"
3"
2 5/8"
2-56 screw holes for vortex valve mounting
Steel plenum chamber and pipe bushing
1/8" mounting studs for vortex valve

FIG. 3

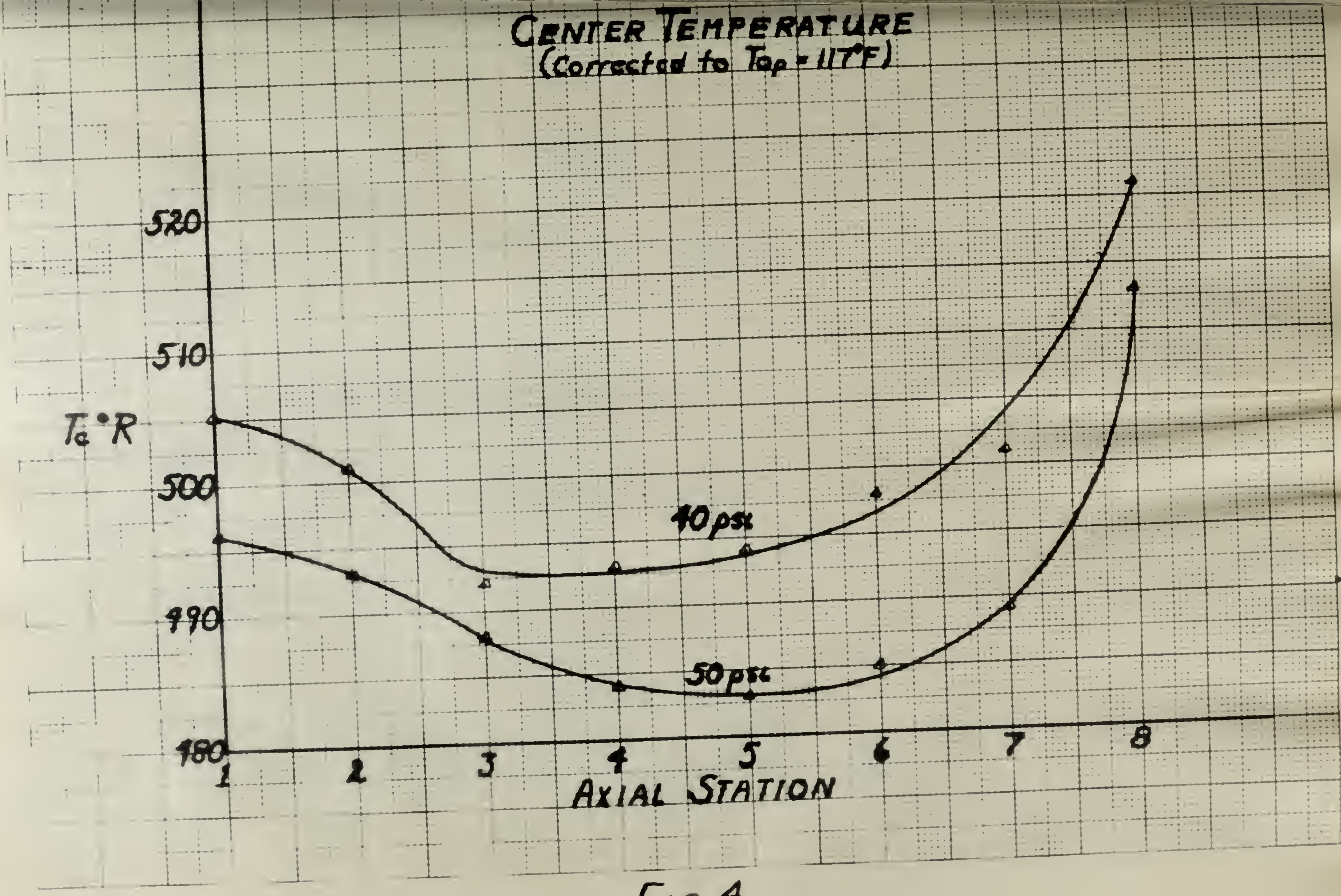

CENTER TEMPERATURE
(Corrected to T_{ap} = 117°F)
T_c °R
520
510
500
490
480
1
2
3
4
5
6
7
8
AXIAL STATION
40 psi
50 psi

FIG. 4

(Corrected to $T_{op} = 118$°F)

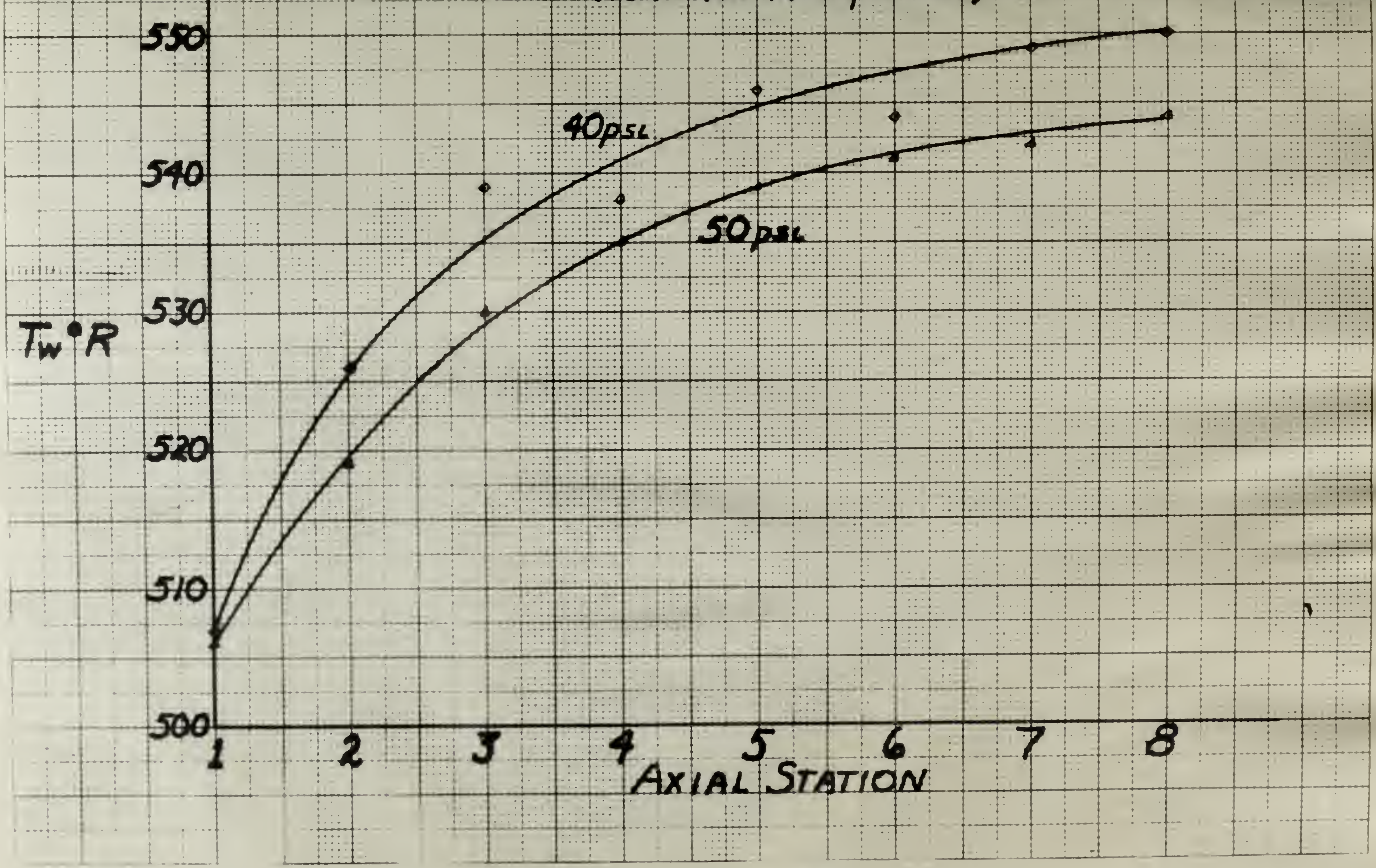


FIG. 5

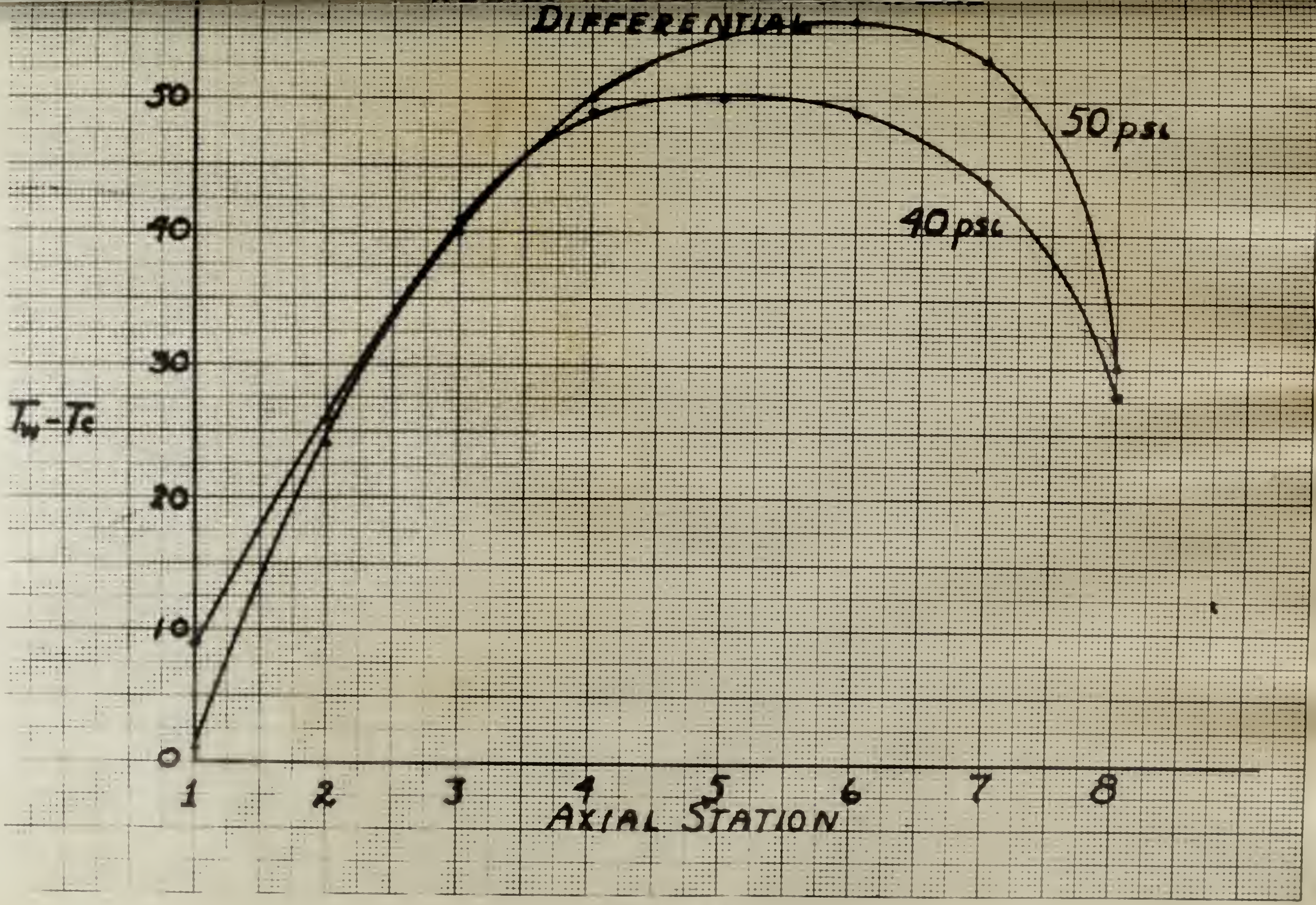
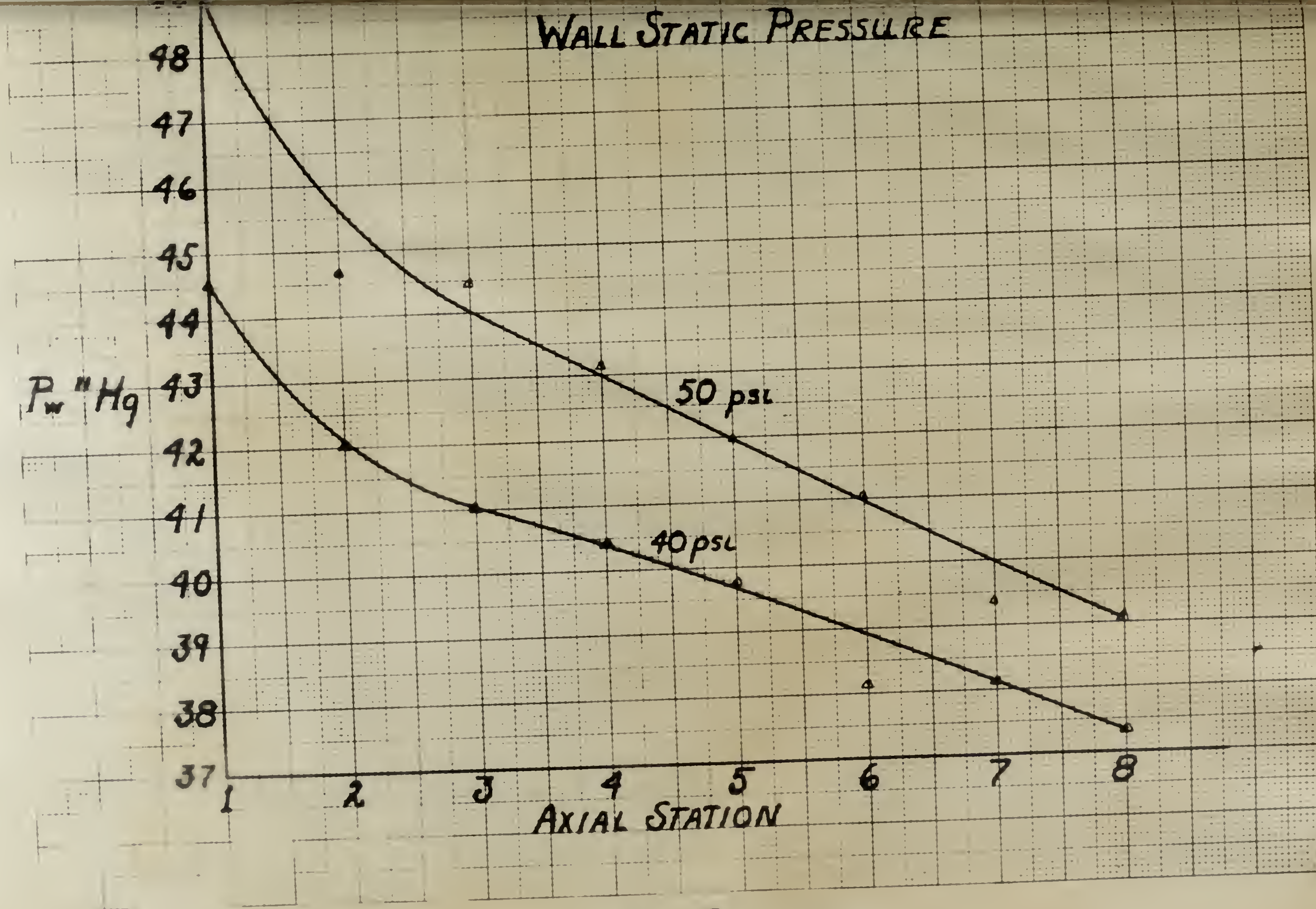
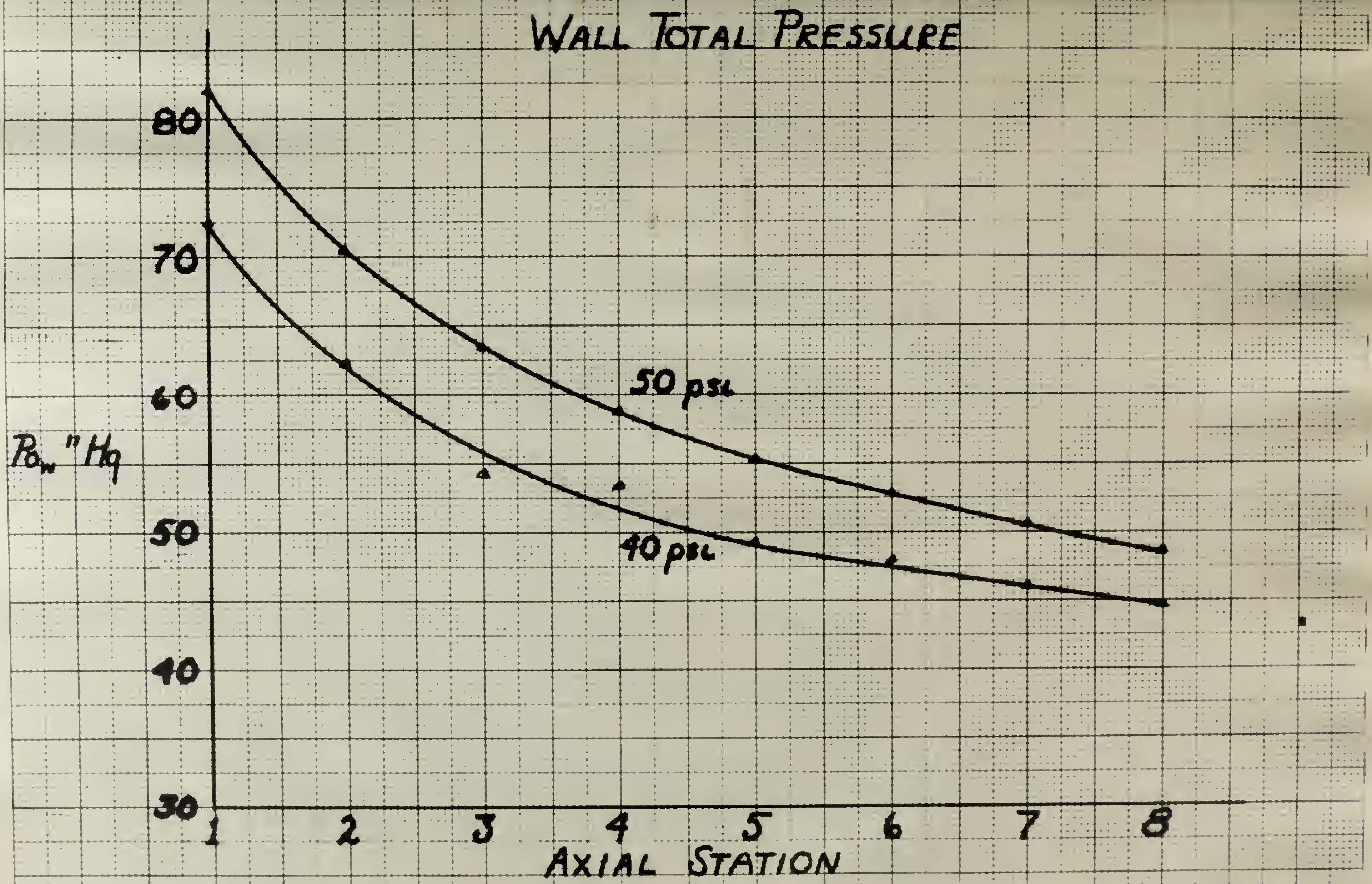

DIFFERENTIAL
$T_w - T_c$
50
40
30
20
10
0
1
2
3
4
5
6
7
8
AXIAL STATION
50 psi
40 psi

FIG 6



WALL STATIC PRESSURE

FIG. 7


WALL TOTAL PRESSURE
P_{0_w} "Hg
80
70
60
50
40
30
50 psi
40 psi
1
2
3
4
5
6
7
8
AXIAL STATION

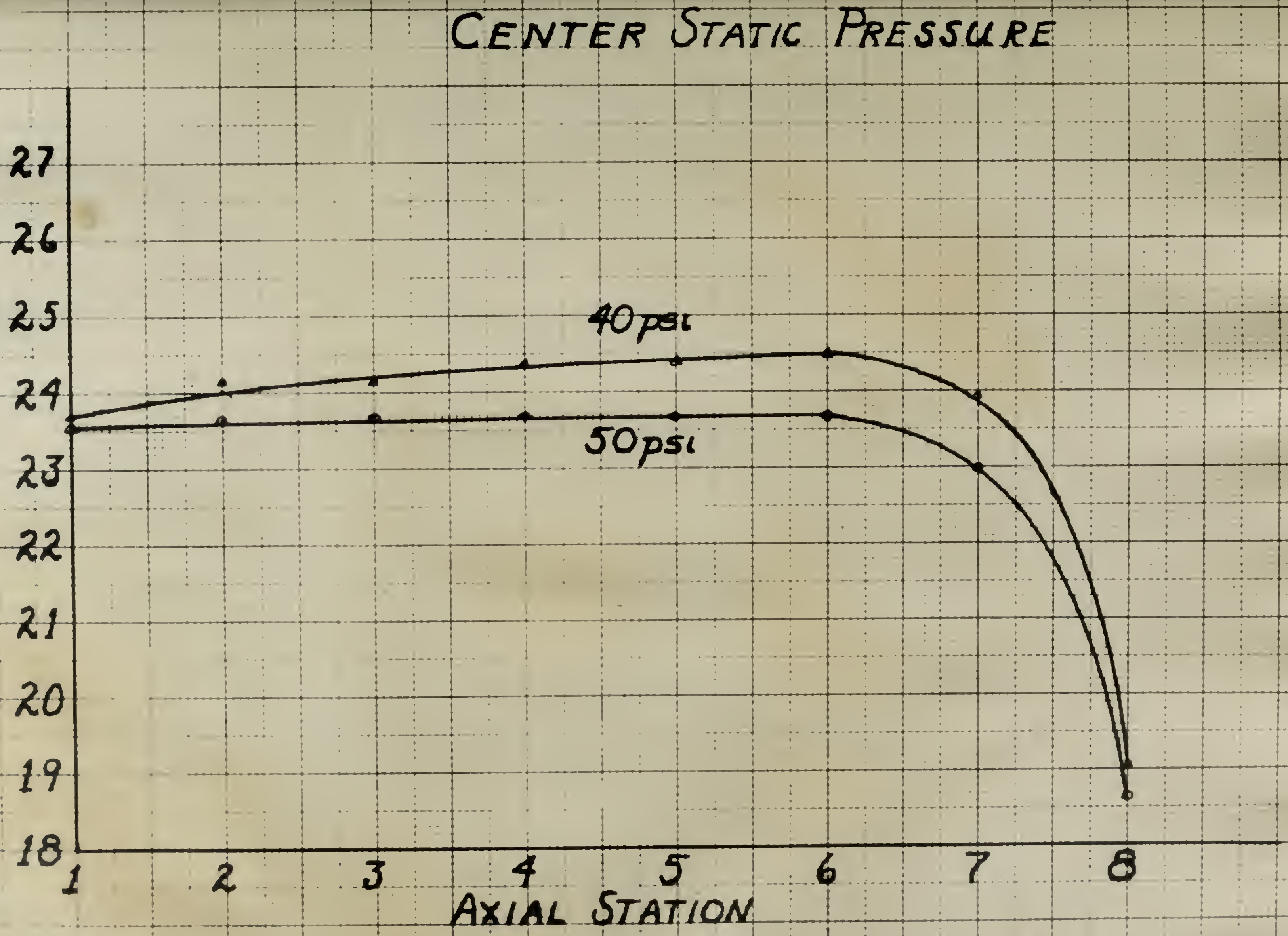
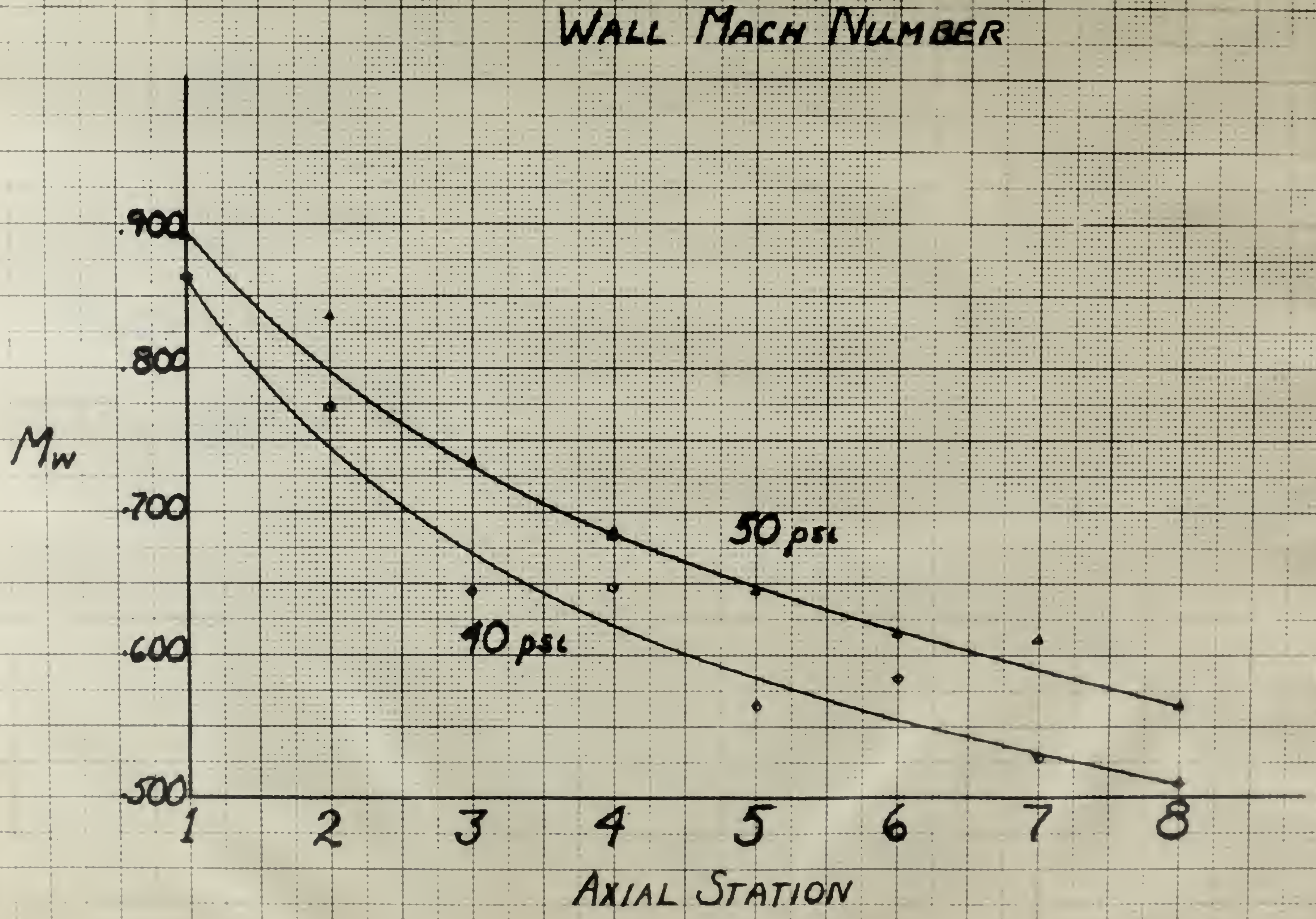

CENTER STATIC PRESSURE
27
26
25
24
23
22
21
20
19
18
40psi
50psi
1
2
3
4
5
6
7
8
AXIAL STATION

Fig. 9


WALL MACH NUMBER

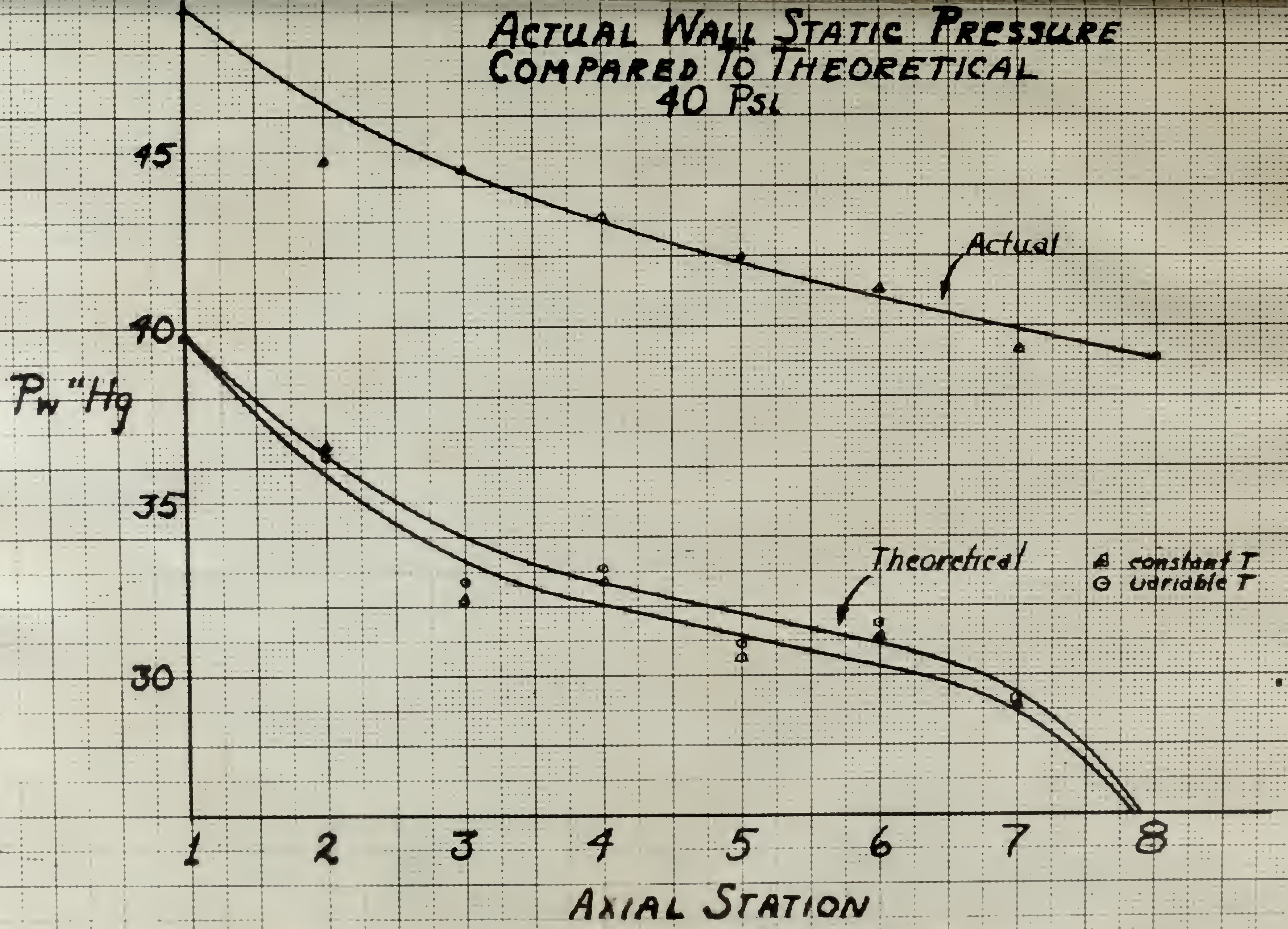
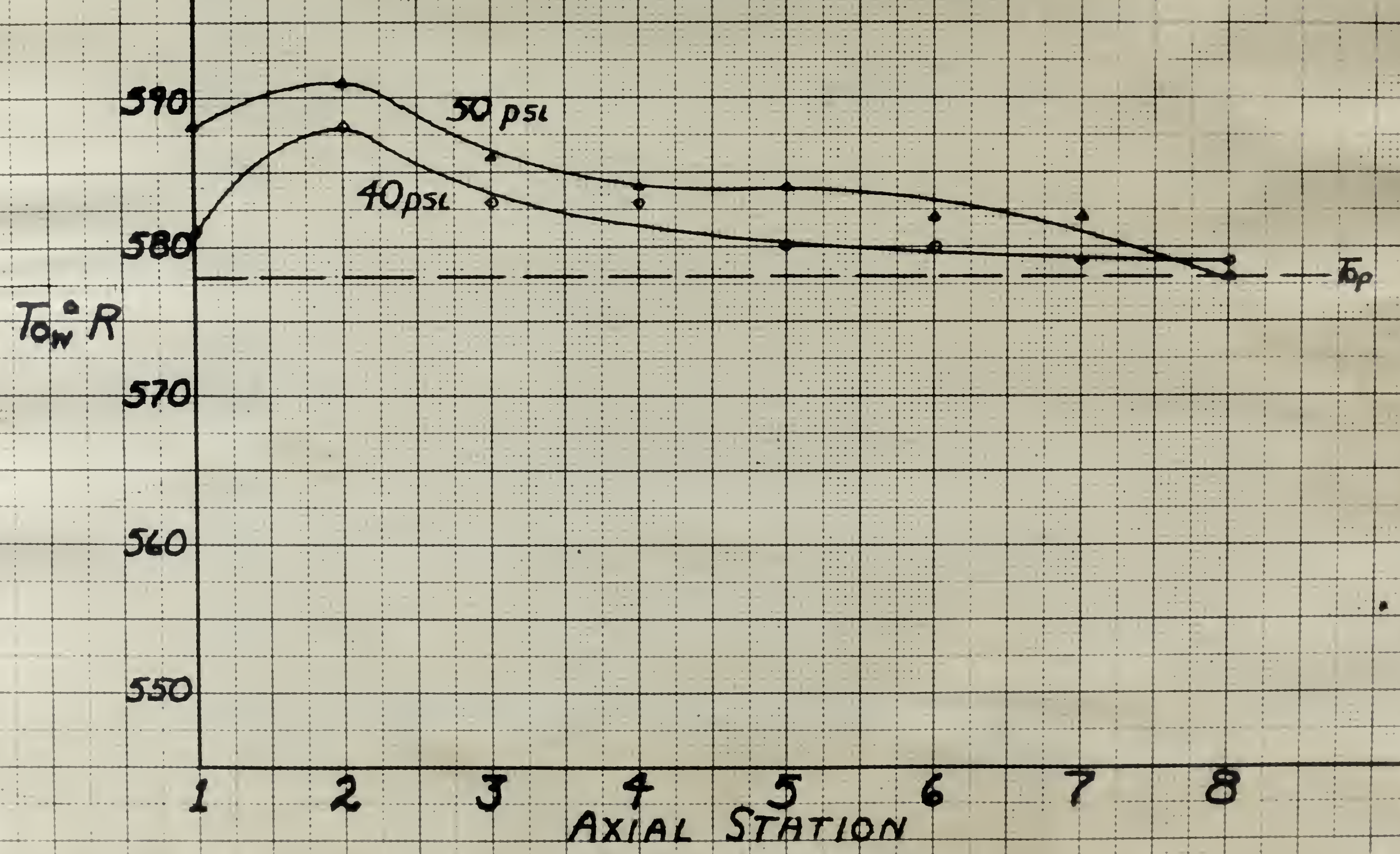

ACTUAL WALL STATIC PRESSURE
COMPARED TO THEORETICAL
40 PSI

FIG. 11

WALL TOTAL TEMPERATURE

(Corrected to $T_{0p} = 118°F$)



DIFFERENTIAL

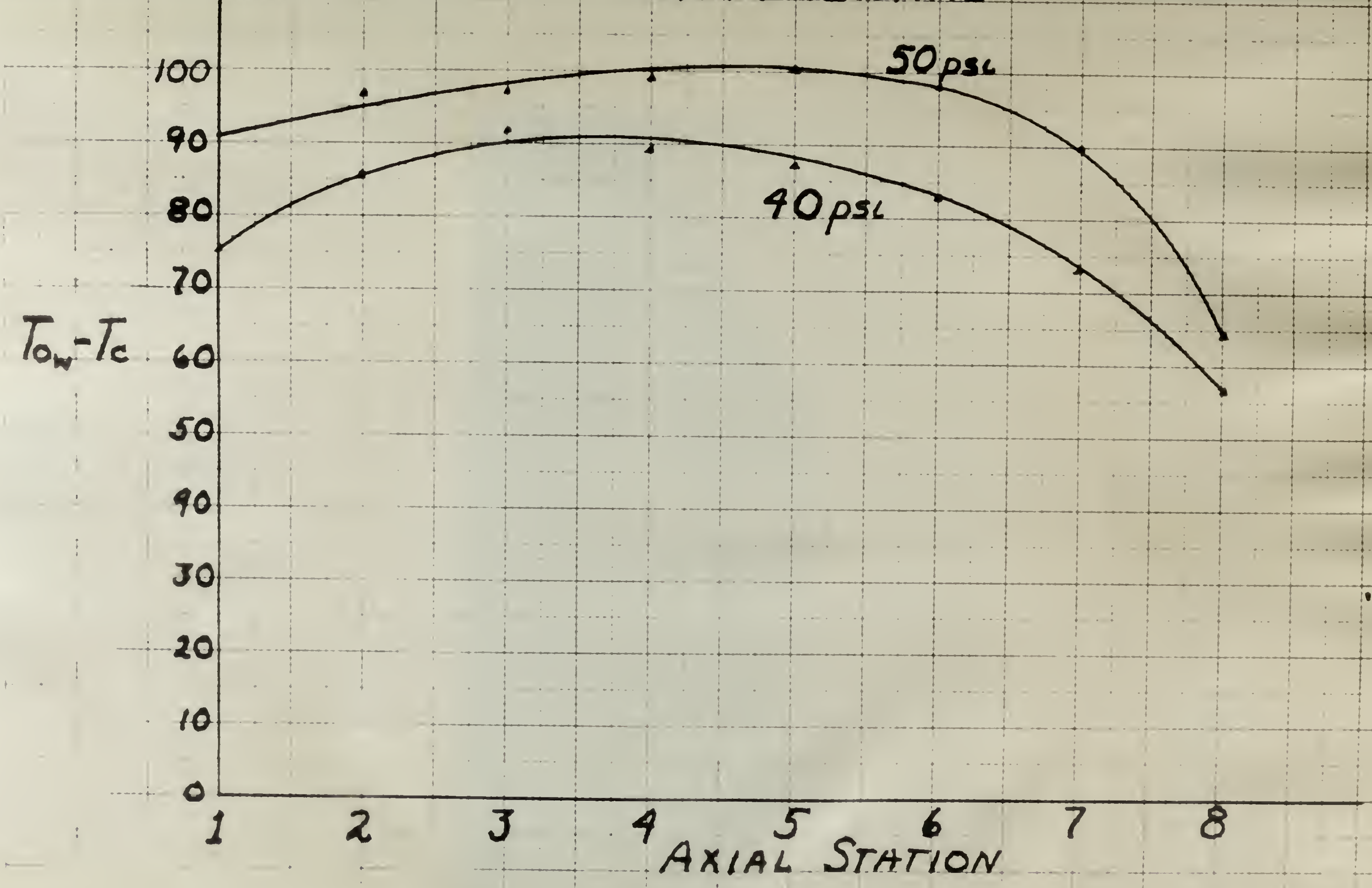


FIG 13

UNMOUNTED VORTEX VALVE
AND TUBE

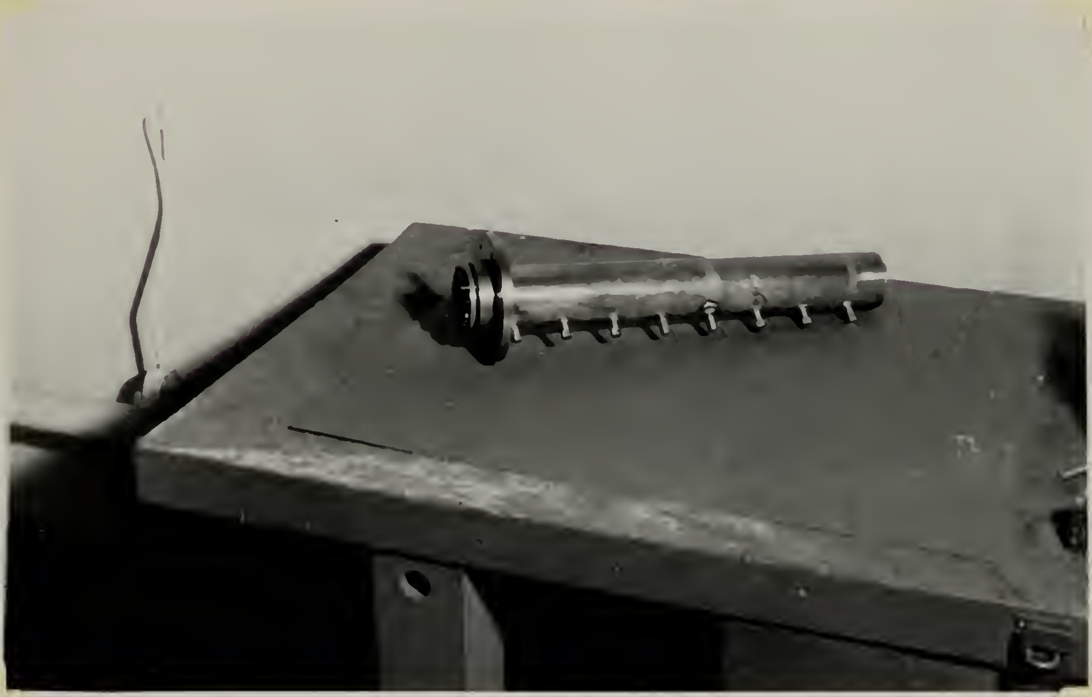


FIG. 14

MOUNTED VORTEX VALVE, TUBE AND CONE

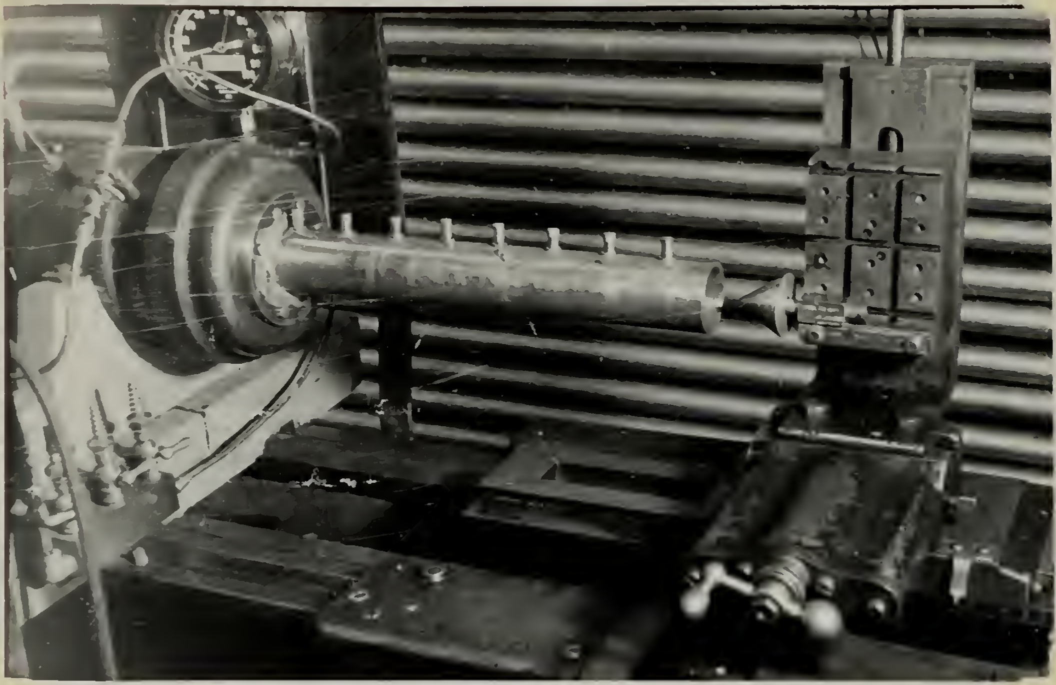


FIG. 15

WALL PROBES AND COLD JUNCTION

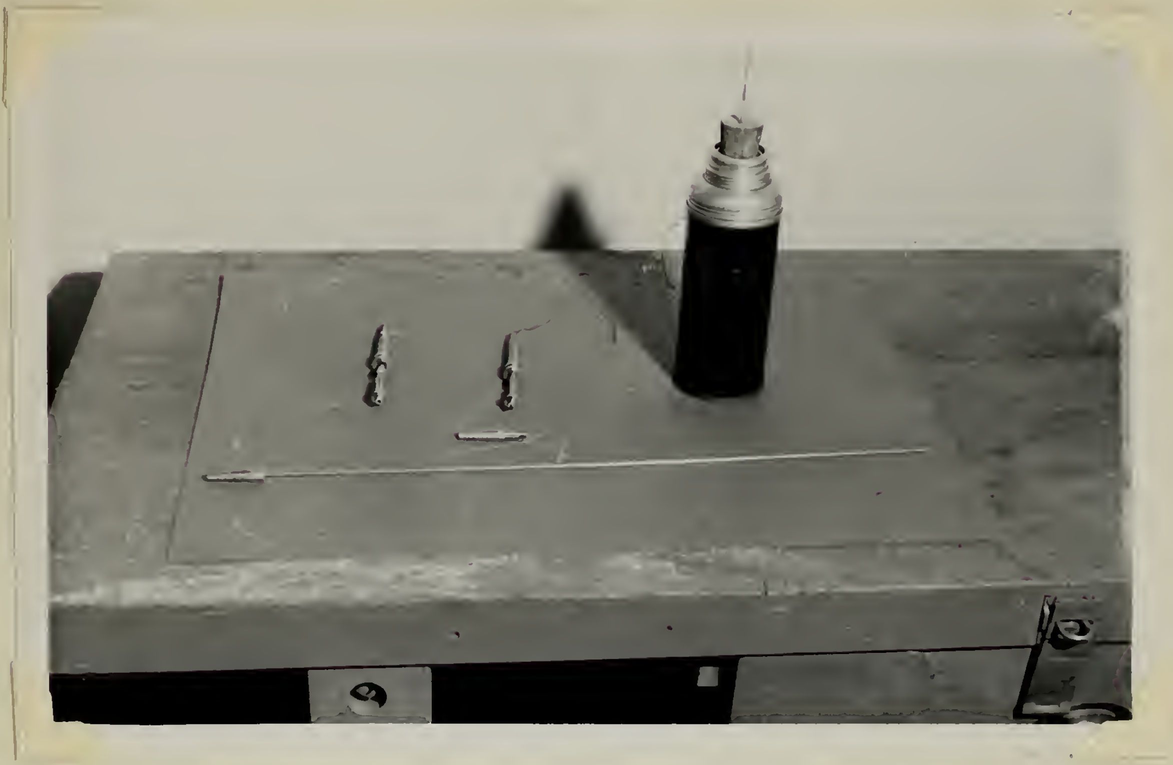


FIG. 16

CENTER PROBE

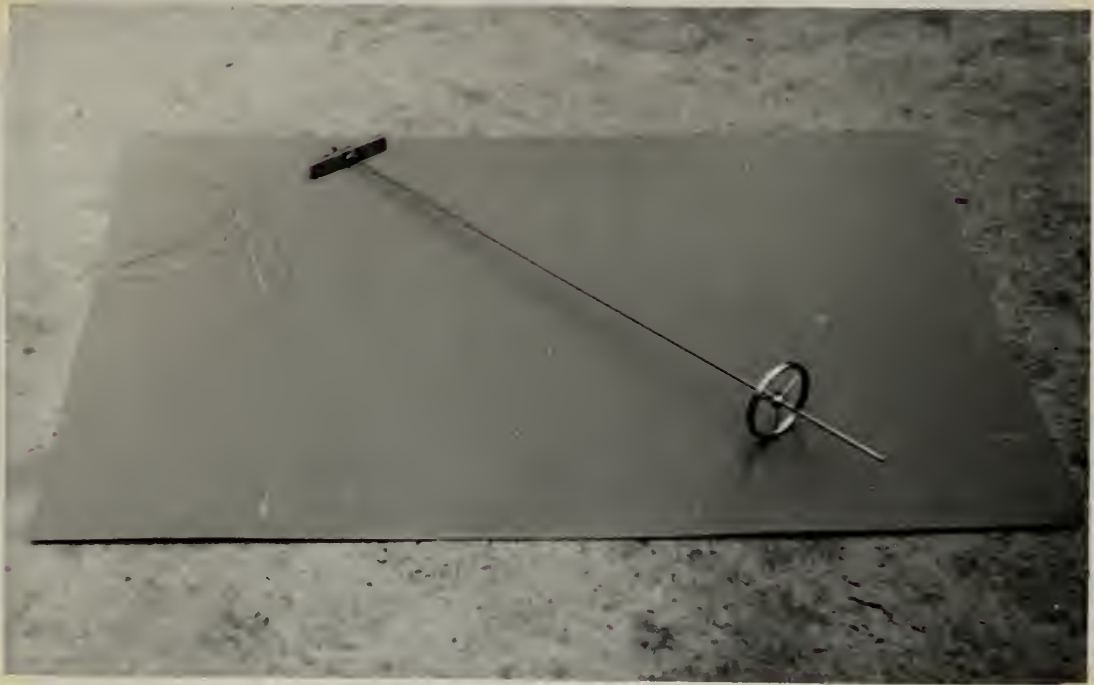
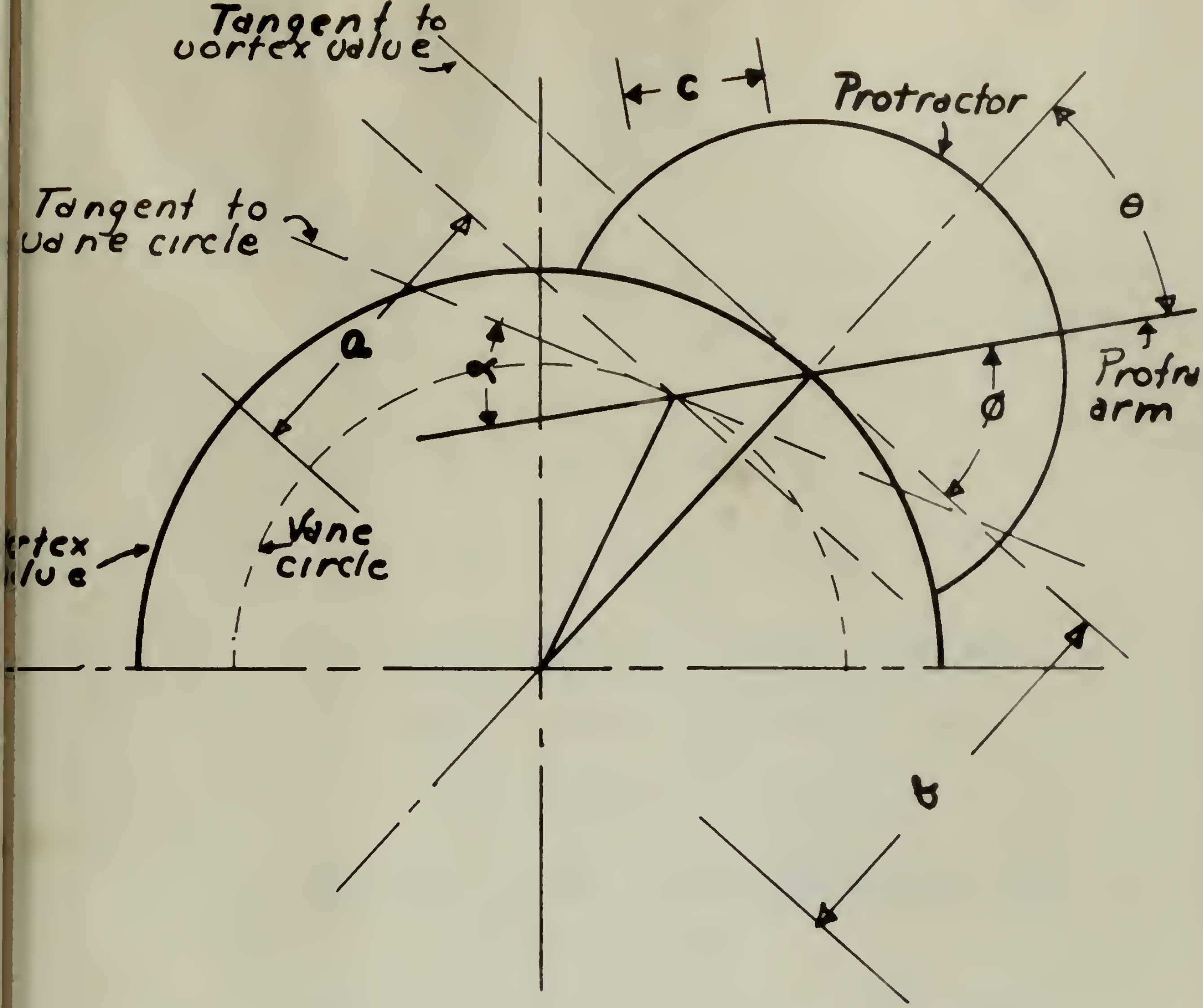


FIG. 17

APPENDIX A

This theoretical development serves merely as a convenient reference for other investigators using the vortex valve employed by this author. In the following pages and on the following figure are developed the equations necessary to use the protractor for setting the vortex vanes and a plot of the vane settings with respect to the vane circle for each protractor setting.



α is the vane angle with respect to a vane circle tangent

θ is angle between reference line and protractor arm

a is the radius of the vane circle

b is the radius of the vortex valve

$\frac{\sin A}{a} = \frac{\sin B}{b}$ where A & B are the angles opposite a & b

$$\sin B = \frac{b}{a} \sin A$$

$$B = \sin^{-1}\left(\frac{b}{a} \sin A\right)$$

$$C = 180 - (B + A) = 180 - (B + \theta)$$

$$C = 180 - \sin^{-1}\left(\frac{b}{a} \sin\theta\right) - \theta$$

$$C = \phi - \alpha \qquad \alpha = \phi - C$$

$$\alpha = \phi - 180 + \sin^{-1}\left(\frac{b}{a} \sin\theta\right) + \theta$$

$$\phi = 90 - \theta$$

$$\alpha = 90 - \theta - 180 + \sin^{-1}\left(\frac{b}{a} \sin\theta\right) + \theta$$

$$\alpha = -90 + \sin^{-1}\left(\frac{b}{a} \sin\theta\right)$$

$$b = 1\tfrac{3}{8}'' \qquad a = 1\tfrac{1}{16}''$$

$$\frac{b}{a} = 1.293$$

$$\alpha = -90 + \sin^{-1}(1.293 \sin\theta)$$

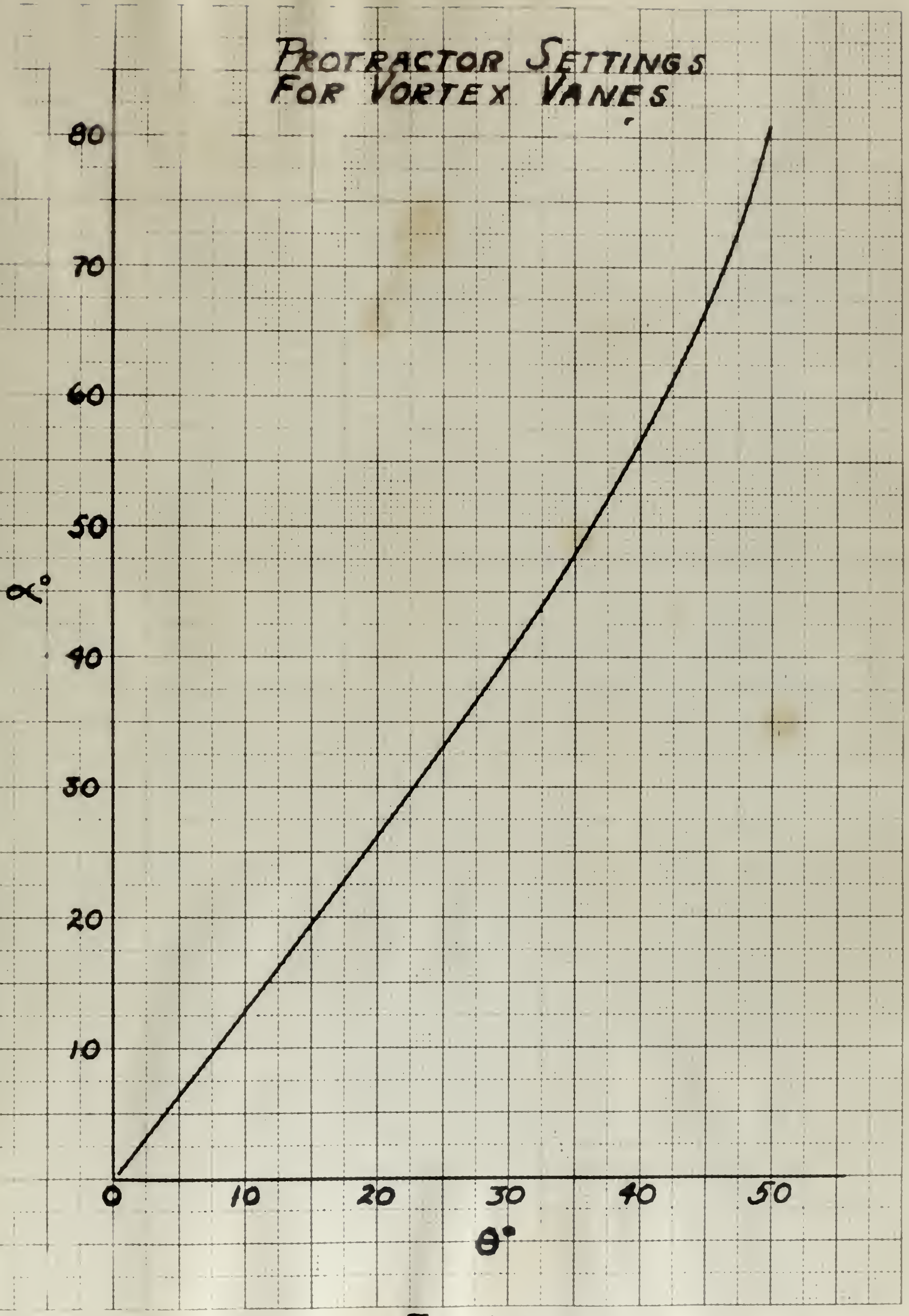

PROTRACTOR SETTINGS
FOR VORTEX VANES
80
70
60
50
α°
40
30
20
10
0
10
20
30
40
50
θ°

FIG. 1-A

APPENDIX B

The investigation in Appendix B was conducted with the vortex valve but without utilizing the vortex tube. As can be seen from Figure 1-B, the vortex valve was secured to aluminum housing mounted directly on the plenum chamber. Air then flowed from the plenum chamber through the vortex vanes expanding radially inward with a high angular velocity and discharging through a one half inch orifice in the face of the housing.

Instrumentation holes were provided radially from the orifice starting one tenth of an inch from the orifice with two tenths of an inch between centers. Four instrument holes were used with the last hole being one eighth of an inch in from the vortex vane circle.

The instruments used and their calibration were furnished by Major D. H. Heaton, USAF. The total pressure probe and temperature probe were made from number 20 stainless steel hypodermic needles, and the static pressure probe was merely a tapped stud fitting into the instrumentation hole at the desired station.

During the run, a pressure of thirty pounds per square inch gage was maintained in the plenum

chamber. This pressure resulted in a Mach number at the discharge of approximately .910.

In all readings, the total temperature and pressure were taken in the center of the radial section at each instrumentation hole. It was assumed that the static pressure was constant at any given radius.

From Figure 2-B it can be seen that the product of Mach number and radius is almost a constant. As the Mach number is approximately proportional to the velocity in this investigation, the conditions existing are very nearly those of a true mathematical free vortex. This is substantiated by the essentially constant total temperature in a radial direction.

Barometer 29.99 "Hg

Top 110° F

Station	Po "Hg	Po "Hg	P "Hg	P "Hg
1	57.3	87.29	37	66.99
2	57.3	87.29	48.3	78.29
3	60.3	90.29	54.0	83.99
4	59.6	89.59	56.5	86.49

Po/P	M	To(MV)	To$_{instr.}$	To$_{correct}$
1.304	.630	1.97	100	110
1.116	.400	2.09	105	111
1.075	.324	2.10	105	110
1.037	.228	2.10	105	109

*Stations numbered from nozzle out.

TABLE 1-B

VORTEX VALVE MOUNTING

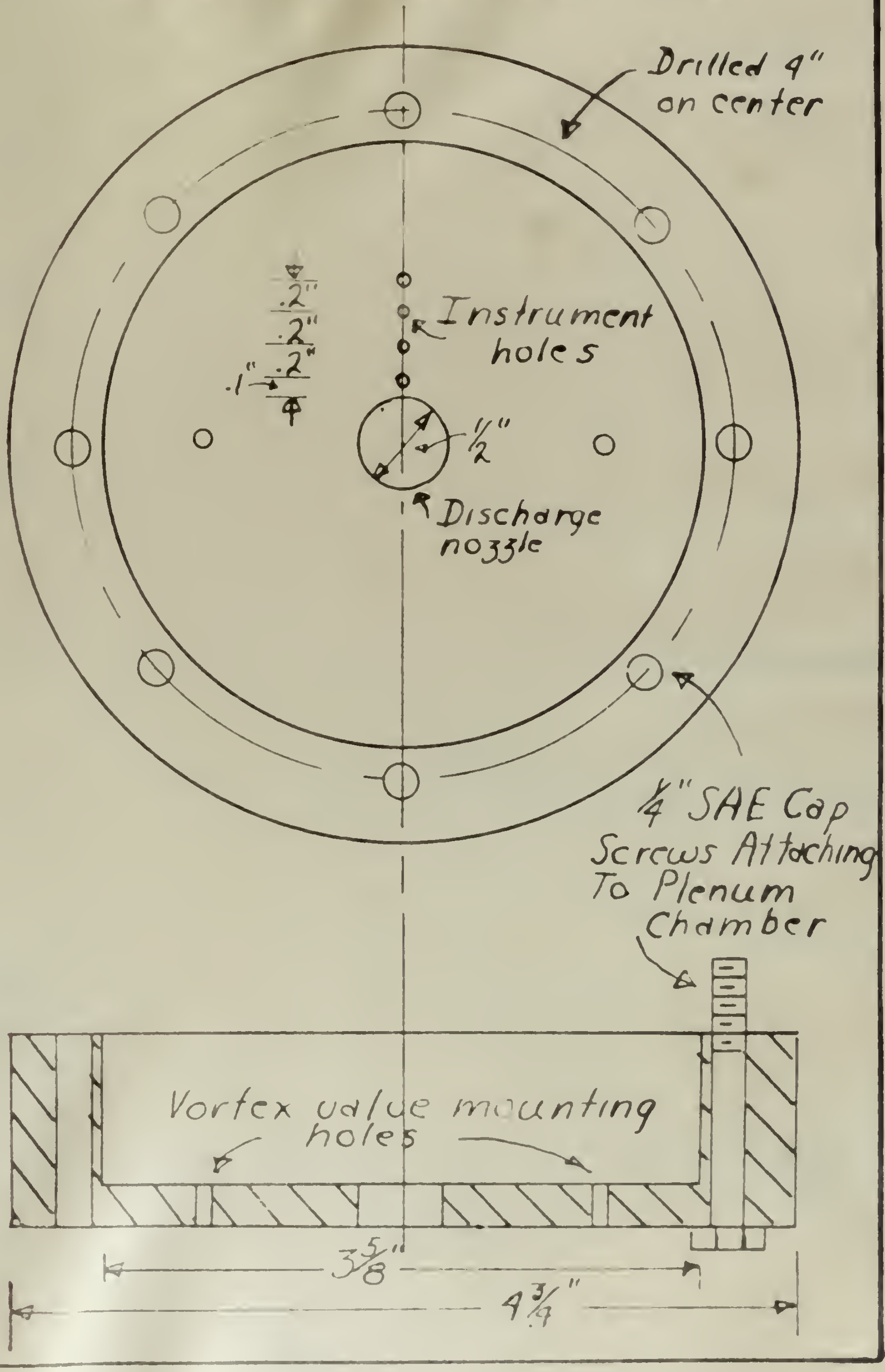


FIG 1-B

RADIAL DISTRIBUTION OF
MACH NUMBER AND PRODUCT
OF MACH NUMBER AND RADIUS

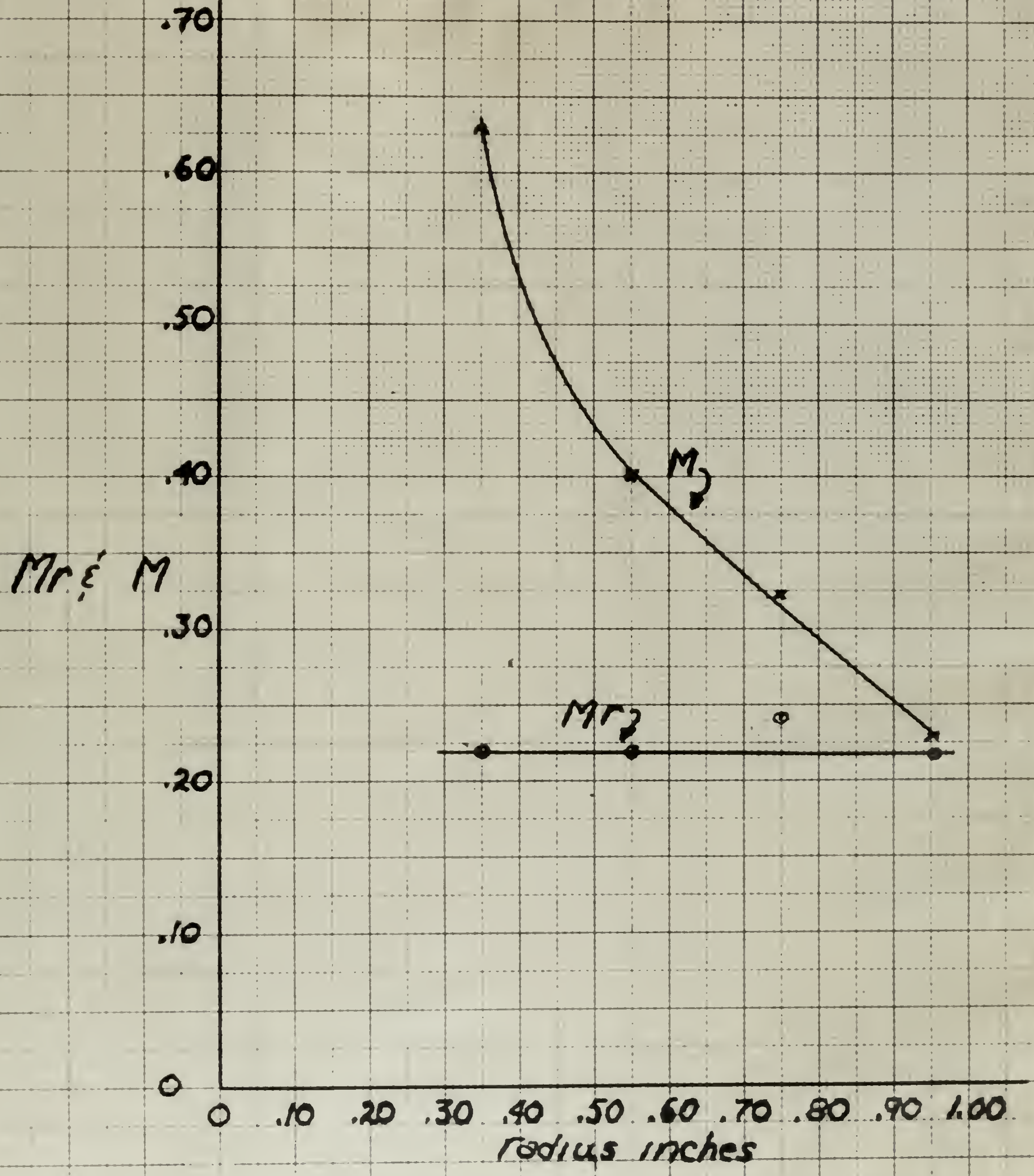


FIG. 2-B

APPENDIX C

The investigation in Appendix C was to attempt an approximate determination of whether or not diffusion was taking place in the vortex tube. Although it was admitted in advance that the equipment readily available for making this determination would be crude and that the two main component gases, oxygen and nitrogen, had nearly the same molecular weights, it was felt that there was some chance of getting at least an indication on which future more careful investigations could be conducted.

Standard Orsat equipment was utilized, taking samples from station number five of the vortex tube. A double valved manifold was used to reduce the existing pressure and provide a fine control over sampling. The plenum chamber was maintained at 40 pounds per square inch gage during the sampling.

Three determinations were made of the free air in the laboratory and of the air at the periphery of the vortex tube and no significant results were obtained. This should not be interpreted as ruling out any such possibility but merely reflects on the accuracy of the employment of apparatus beyond its design limitations. It would be interesting to see what results would be obtained by using mixtures of widely different molecular weight gases and more accurate quantitative measuring devices.

DATE DUE

hin

12835
Thesis
H96 Hutchin
Energy transformation
in a vortex tube.

Thesis 12835
H96 Hutchin
Energy transformation
in a vortex tube.

thesH96
Energy transformation in a vortex tube.
3 2768 002 13296 1
DUDLEY KNOX LIBRARY